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# INTEREST RATE RISK, PREPAYMENT RISK, DURATION RISK, AND A NEW MORTGAGE 

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#### Abstract

Interest rate risk is a key factor in the pricing of fixed-income (debt) securities. When interest rates rise, the value of a fixed-income security falls; when interest rates fall, the value of a fixed-income security rises. However, when interest rates fall, many issuers of fixed-income securities "call" their outstanding debt securities in order to refinance at lower rates of interest. Buyers seek to protect against this event by demanding a "call premium" or other penalty should the debt be "called."

In the US, a very high percentage of conventional residential fixed-rate mortgages (FRMs) and virtually all government insured and guaranteed residential FRMs are sold into the secondary mortgage market (SMM) where these mortgages are bundled together and mortgage-backed securities, such as collateralized mortgage obligations, are issued against them. Unfortunately for the buyers of US FRMs, virtually all these mortgages may be prepaid in whole or in part at any time without penalty.

The problem of interest rate risk is exacerbated by the asymmetry in prepayments given a change in interest rates. When interest rates fall, US mortgagors' prepayments tend to accelerate as refinancings and house sales increase; when interest rates rise, prepayments decline as US mortgagors seek to hold on to "below market rate" financing for as long as possible. Thus, buyers of US residential mortgages face an uncertain expected duration of their investments - realizing a longer-than-desired duration when interest rates rise and a shorter-than-desired duration when interest rates fall. This uncertainty manifests itself in a risk premium that lowers the price SMM buyers are willing to pay for US residential FRMs, and, by translation, raises the interest rate charged on newly issued FRMs.

Beginning two decades ago, adjustable-rate mortgages (ARMs) became the preferred solution to this problem. ARMs allow the mortgagee to periodically adjust the rate of interest charged in response to changes in the general level of interest rates. Although ARMs are popular when interest rates are at historically high levels (and thus likely to decline), ARMs are thoroughly unpopular with mortgagors when interest rates are low. And, in a rising interest rate environment, default rates on ARMs tend to be higher than on FRMs.

This paper presents an alternative mortgage which retains the fixed-rate feature of an FRM, but accelerates the principal amortization when interest rates rise, exposing the buyer to less duration risk in a rising interest rate environment. This mortgage, labeled the Adjustable Amortization Mortgage (or AAM), is shown to have lessened interest rate risk for the buyer as well as lower default risk, suggesting that it should be priced higher (at a lower rate of interest) than the typical FRM. It is also shown that mortgage-backed securities collateralized by an AAM have much less price volatility than mortgage-backed securities backed by FRMs.


## KEYWORDS:

Mortgages, real estate finance, secondary mortgage market

# Interest Rate Risk, Prepayment Risk, Duration Risk, and a New Mortgage 

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## Introduction

A very large percentage of intermediate-to-long-term (15 years and longer) residential and commercial mortgages in the United States are sold into the secondary mortgage market (SMM). These mortgages are then bundled together, usually by credit quality, interest rate and term, and securities are sold using the cash flow from the bundled mortgages as the basis for payment. Although a variety of securities may be sold, one of the most common is the collateralized mortgage obligation (CMO) which is divided into separate slices or tranches each with a different priority on the cash flow from the underlying mortgages.

Buyers of CMO tranches face a variety of risk factors - among them interest rate risk, prepayment risk, and duration risk. Although CMO writers may take a variety of steps to attempt to reduce these risk factors, such as creating planned amortization class (PAC) tranches, pooling adjustable-rate mortgages, and the like, these risk factors cannot be eliminated entirely. In fact, prepayment risk (the risk that prepayments by mortgagors will differ from what is expected) remains a telling problem for CMO writers since virtually all residential mortgages may be prepaid in whole or in part at any time without penalty.

Prepayment risk is exacerbated by the fact that prepayments tend to accelerate as interest rates decline (home sales and refinancings increase), and to decelerate as interest rates rise (home sales and refinancings decrease). Thus, the duration of a CMO tranche investment tends to move in the same direction as interest rates. This is an especially acute problem when interest rates rise. In this environment, CMO investors see a decline in the value of their investments in two ways: the increase in interest rates reduces the present value of future interest and principal cash flows; and, as prepayments decline, duration increases - further diminishing the value of expected cash flows.

In this paper, a new type of fixed-rate mortgage - the adjustable amortization mortgage (AAM) - is proposed as a partial solution to the prepayment risk/duration risk problem faced by CMO investors. Like a standard fixed-rate mortgage, the interest rate on an AAM is fixed for the term of the mortgage; unlike a standard fixed-rate mortgage, the payments (and, hence, the amortization of principal) increase in response to a rise in the general level of interest rates.

It is shown that CMOs (and their respective tranches) which are written against pools of adjustable amortization mortgages have much lower levels of prepayment risk and duration risk
in a rising interest rate environment. Thus, owners of AAMs and the securities written against them have less downward price volatility. Ceteris paribus, AAMs and securities written against them should command premium prices (lower yields) than corresponding standard fixed-rate mortgages and securities written against them.

## Prior Research

The effects of interest rate changes, prepayments and duration on the pricing of mortgages has been the subject of widespread research. Kau, Keenan, Muller and Epperson (1990, 1993) develop a contingent claims model for mortgage pricing based on the interest rate model of Cox, Ingersoll and Ross (1985). Kau, et al., show that the prepayment option may be of great value to the borrower under a fixed-rate mortgage and, hence, an important component in the pricing of FRMs. Hendershott and Shilling (1985), Ott (1986) and Tucker (1991) all look at various interest rate risk pricing issues while Haensly, Waller and Springer (1993) focus on duration and price behavior in fixed-rate mortgages.

Thode (1991) develops a fixed payment, adjustable-rate mortgage and demonstrates that it has less default risk and less interest rate risk than a standard ARM. Thode and Kish (1993, 1994) construct a fixed-rate mortgage that uses a prepaid zero coupon treasury bond for principal repayment. They show that this mortgage has less prepayment (and duration) risk than a standard FRM.

Numerous studies have explored the prepayment patterns of ARMs and FRMs and the effects of prepayments on the valuation of mortgage-backed securities. These include Anderson, Barber and Chang (1993) who focus on the effects of prepayments on duration, and McConnell and Singh (1991), and Richard and Roll (1989) who evaluate interest-dependent prepayments and their effects on risk. McConnell and Singh, and Richard and Roll note that interestdependent prepayments are most significant in the early years of a mortgage, and more significant in the pricing of fixed-rate mortgages than in adjustable-rate mortgages.

## Duration

Measurement of the price elasticity of a stream of fixed cash flows can be accomplished through the statistic known as duration, which was first proposed by Macauley (1938). Fundamentally, duration seeks to measure the effective maturity of a stream of cash flows. In Macauley's formulation, the weight of each receipt is related to the importance of the receipt in a present value context.

Although Macauley's original formulation has been refined and modified through time (e.g., Anderson, Barber and Chang [1993]), authors such as Bierwag, Kaufman and Toevs (1983) show that Macauley's measure performs quite well against latter formulations.

The general formulation of Macauley duration is given by:

$$
\begin{align*}
& \text { DUR }=\searrow \frac{\mathrm{t}^{*} \mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{r})^{t}} / \mathrm{m} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{r})^{t}}  \tag{1}\\
& \mathrm{t}=1 \\
& \mathrm{t}=1
\end{align*}
$$

Where: CF $_{\mathrm{t}} \quad=$ Payment (cash flow) the security will make in time period t ;
$\mathrm{N} \quad=$ Number of time periods until the maturity of the security;
r = Security's required return (per period); and,
$\mathrm{m} \quad=$ Number of periods per year. ${ }^{1}$
If we define modified duration $\left(\mathrm{DUR}_{\text {mod }}\right)$ as $\mathrm{DUR} /(1+\mathrm{r})$, then modified duration multiplied by a given change in (annualized) yield (or required return) approximates for the negative of the expected percentage price change or return on a security:
$\mathrm{DUR}_{\mathrm{mod}} * \rightarrow_{\mathrm{r}}{ }^{*} \quad \not \subset-\rightarrow$ PRICE/PRICE
m

Duration is an absolute measure of interest-rate risk and is particularly applicable to fixedpayment securities. It can then be applied to compare the price sensitivity of different securities. However, as will be noted later, equation 2 approximates for the negative of the expected percentage security price change only if there is no change in the expected cash flows each period $\left(\mathrm{CF}_{\mathrm{t}}\right)$ as a result of a change in interest rates (required returns). ${ }^{2}$

## CMO Valuation and Risk

For any given CMO backed by a pool of fixed-rate mortgages, the market value of a particular tranche $\mathrm{j}\left(\mathrm{V}_{\mathrm{j}}\right)$ may be modeled as:

$$
\begin{equation*}
V_{j}=f\left(C_{j}, F_{j}, r_{j}, T_{j}\right) \tag{3}
\end{equation*}
$$

Where: $\mathrm{C}_{\mathrm{j}} \quad=$ Periodic coupon interest paid to tranche j ;
$\mathrm{F}_{\mathrm{j}} \quad=$ Face value principal amount of tranche j ;
$r_{j} \quad=$ Required rate of return on investment for tranche $j$; and,
$\mathrm{T}_{\mathrm{j}} \quad=$ Expected term to maturity for tranche j .
The value of tranche $j$ at any given point time $t(t=0)$ is equal to the present value of the future cash flows received over N time periods, or:

## N

[^0]Where: $\mathrm{P}_{\mathrm{j}, \mathrm{t}} \quad=\quad$ Principal payments to tranche j during time period t

$$
\left(\mathrm{C}_{\mathrm{j}, \mathrm{t}}+\mathrm{P}_{\mathrm{j}, \mathrm{t}}\right)=\stackrel{\left(\boxtimes \mathrm{P}_{\mathrm{j}, \mathrm{t}}=\mathrm{F}_{\mathrm{j}}\right) ; \text { and, }}{\mathrm{CF}_{\mathrm{j}, \mathrm{t}}^{3}}
$$

In a typical flow-through CMO, principal payments to the tranche consist of the normal, amortizing principal paid by mortgagors each period plus principal prepayments made in whole (as in the sale of the home or a refinancing), or in part (as in the case of a mortgagor who elects to make an additional, partial principal payment).

If the coupon payments $\left(\mathrm{C}_{\mathrm{j}, \mathrm{t}}\right)$ and principal payments $\left(\mathrm{P}_{\mathrm{j}, \mathrm{t}}\right)$ are divided into separate claims, then the value of the interest-only stream to tranche $\mathrm{j}\left(\mathrm{IO}_{\mathrm{j}}\right)$ is given by:

$$
\begin{equation*}
\mathrm{IO}_{\mathrm{j}}={\underset{\mathrm{t}=0}{\mathrm{~N}} \frac{\mathrm{C}_{\mathrm{j}, \mathrm{t}}}{\left(1+\mathrm{r}_{\mathrm{j}}\right)^{t}}}_{\underset{\mathrm{t}}{\mathrm{~N}}} \tag{5}
\end{equation*}
$$

And the value of the principal-only stream to tranche $j\left(\mathrm{PO}_{\mathrm{j}}\right)$ is given by:

$$
\begin{equation*}
\mathrm{PO}_{\mathrm{j}}={\underset{\mathrm{t}=0}{\mathrm{~N}} \frac{\mathrm{P}_{\mathrm{j}, \mathrm{t}}}{\left(1+\mathrm{r}_{\mathrm{j}}\right)^{t}}}_{\underset{\mathrm{t}}{\mathrm{~N}}} \tag{6}
\end{equation*}
$$

Thus, $\mathrm{V}_{\mathrm{j}}=\mathrm{IO}_{\mathrm{j}}+\mathrm{PO}_{\mathrm{j}}$.
Therefore, the total value of the cash flows to a CMO tranche depends on the timing and amount of the interest (coupon) and principal payments, and the discount rate applied to these cash flows. In turn, the discount rate (required rate of return) may be modeled as a function of the riskless rate of interest $\left(\mathrm{r}_{\mathrm{f}}\right)$ plus a risk premium (RP), or:

$$
\begin{equation*}
r_{j}=r_{f}+R P_{j} \tag{7}
\end{equation*}
$$

Where: $\mathrm{RP}_{\mathrm{j}}=$ risk premium appropriate for the cash flows to tranche j .

[^1]The risk premium is a function of a variety of risk factors including the default risk of the underlying mortgages, and the expected duration of the cash flows to the tranche (which is affected by the expected rate of principal prepayments on the underlying mortgages).

## Prepayment Rates on Fixed-Rate Mortgages

Unlike other forms of debt, the vast majority of residential mortgages may be prepaid in whole or in part by the mortgagor at any time without penalty. In the absence of penalties, the main factors affecting the rate of prepayments are: 1) housing turnover (home sales); 2) refinancings; and, 3) involuntary prepayments due to death, divorce, foreclosure, etc. [see Bartlett, 1994]. Home sales are the prime factor in prepayments when an existing mortgage is selling at a discount; refinancings are the prime factor in prepayments when an existing mortgage is selling at a premium.

Of critical importance to the mortgagee and the owners of securities written against mortgages is the effect of prepayments on the market values of their holdings. In general, prepayments tend to moderate the increase in mortgage and mortgage-backed security prices when interest rates fall. Falling interest rates tend to accelerate prepayments both because housing turnover tends to increase (more prospective homebuyers qualify for mortgages, increasing housing demand) and because refinancing activity increases (existing mortgagors refinance at lower rates of interest) ${ }^{4}$. Thus, the upward price movement in a mortgage due to a drop in interest rates is constrained by a decrease in the anticipated duration of the cash flows.

When interest rates rise, prepayments decline because housing turnover decreases, and fewer refinancings occur. As a result, the anticipated duration of the cash flows increases, placing additional downward price pressure on mortgages and mortgage-backed securities. ${ }^{5}$

Thus, the rate of prepayment may be modeled as a function of several variables:

```
    PPR = f(I, HT, RE, DF, Other)
Where:PPR = Prepayment rate;
    I = General level of interest rates
    HT = Housing turnover
    RE = Level of refinancing activity;
    DF = Defaults on mortgages; and,
    Other = Other factors (such as divorce, death, etc.) which are assumed to be
                        constant.
```

[^2]In general, $\rightarrow \mathrm{PPR} / \rightarrow \mathrm{I}<0 ; \rightarrow \mathrm{PPR} / \rightarrow \mathrm{HT}>0 ; \rightarrow \mathrm{PPR} / \rightarrow \mathrm{RE}>0 ;$ and, $\rightarrow \mathrm{PPR} / \rightarrow \mathrm{DF}>0$. Since housing turnover, refinancings and defaults are also affected by changes in interest rates, the partial derivatives are: $\partial \mathrm{HT} / \partial \mathrm{I}<0 ; \partial \mathrm{RE} / \partial \mathrm{I}<0$; and $\partial \mathrm{DF} / \partial \mathrm{I}>0$.

Finally, the duration (DUR) of the cash flows to the mortgagees and owners of mortgagebacked securities is inversely related to the rate of prepayment, hence $\rightarrow \mathrm{DUR} / \rightarrow \mathrm{PPR}<0$. As a result, duration can be expected to decrease when: interest rates fall; refinancings rise; and, defaults rise. Conversely, duration can be expected to increase when: interest rates rise; refinancings fall; and, defaults decline.

## The Adjustable Amortization Mortgage

For a typical constant monthly payment, fixed-rate mortgage with a term of N months, annual interest rate i (monthly interest rate $=\mathrm{i} / 12$ ), and original loan balance $\mathrm{P}_{0}$, the monthly payment R which will amortize the mortgage to a $\$ 0$ balance at the end of time period N is given by:

$$
\begin{equation*}
\mathrm{R}=\quad \mathrm{P}_{0} \quad / \quad \mathrm{X} \frac{1}{(1+[\mathrm{i} / 12])^{t}} \tag{9}
\end{equation*}
$$

Where the divisor on the right-hand side of equation 9 is the present value of an annuity factor. The remaining principal balance on the mortgage after $n$ payments have been made ( $\mathrm{P}_{\mathrm{n}}$ ) is given by:

$$
\mathrm{P}_{\mathrm{n}}=\quad \mathrm{R} \quad * \varliminf_{\mathrm{t}=\mathrm{n}+1}^{\mathrm{N}} \frac{1}{(1+[\mathrm{i} / 12])^{\mathrm{t}-\mathrm{n}} .}
$$

Under most fixed-rate mortgage contracts, if the borrower makes additional principal payments (prepayments) at any point in time $\left(\mathrm{PP}_{\mathrm{n}}\right)$ so that the remaining principal balance ( $\mathrm{P}_{\mathrm{n}}^{*}=$ $\mathrm{P}_{\mathrm{n}}-\mathrm{PP}_{\mathrm{n}}$ ) is lowered, subsequent monthly payments remain at R , but the term of the mortgage is reduced. However, the borrower is not compelled, at any point in time, to make a payment larger than R .

In a rising interest rate environment, the market values of fixed-rate mortgages (and the securities written against them) decline as the cash flows are discounted at higher rates. Exacerbating the decline in values due to rising interest rates is the likelihood that prepayments (in whole, or in part) will also decline for the reasons stated in the previous section. A decline in prepayments increases the duration of the mortgage, placing additional downward pressure on prices.

For example, given a change in the required rate of return on a CMO tranche $j$ from $r$ to $r^{*}$, the change in the market value of tranche $j$ can be modeled as:


If it is assumed that the prepayment rate on the underlying mortgages is unaffected by the change in required return, then $C_{j, t, r^{*}}=C_{j, t, r}$ and $P_{j, t, r^{*}}=P_{j, t, r} .{ }^{6}$

However, if the prepayment rate changes along with a change in required return, then $\mathrm{C}_{\mathrm{j}, \mathrm{t}, \mathrm{r}^{*}} \gamma \mathrm{C}_{\mathrm{j}, \mathrm{tr}}$ and $\mathrm{P}_{\mathrm{j}, \mathrm{tr}}{ }^{*} \gamma \mathrm{P}_{\mathrm{j}, \mathrm{t}, \mathrm{r}}$. As a result, $\mathrm{CF}_{\mathrm{j}, \mathrm{t}, \mathrm{r}^{*}} \gamma \mathrm{CF}_{\mathrm{j}, \mathrm{tr}}$. Therefore, since duration (and, by translation, modified duration) is affected by the change in the prepayment rate, then the lefthand side of equation 2 will understate the magnitude of the security price change if changes in prepayment rates decline with an increase in required return. The additional change in the price of the security, over and above the approximation given by equation 2 , can be referred to as prepayment risk.

## Constructing an AAM

It is possible to design a fixed-rate mortgage that reduces the buyer's exposure to prepayment risk in a volatile interest rate environment. Such a fixed-rate mortgage is one where the amortization of principal is accelerated in response to an upward change in interest rates. ${ }^{7}$ Similar in construct to an adjustable-rate mortgage (ARM), the adjustable amortization mortgage (AAM) is a fixed-rate mortgage with a (possibly) variable payment. That is, like an ARM, the monthly payment on an AAM is adjusted periodically to reflect changes in the general level of interest rates. However, unlike an ARM, the interest rate on the AAM remains constant throughout its life so that any adjustments in the monthly payment are credited directly against the remaining principal balance of the mortgage.

Rather than transferring interest rate risk (in part or in whole) from the lender to the borrower as is done with an ARM, the AAM transfers prepayment risk from the lender to the borrower. As a result, AAMs (and AAM-backed securities) render some price volatility protection to their owners that fixed-rate mortgages do not offer.

A fairly straightforward AAM is similar in construct to an ARM. The payment on the AAM is adjusted periodically (perhaps annually); there is a well-defined underlying index rate of

[^3]interest upon which changes in the payments are benchmarked; there is a defined margin over the index rate; and, the AAM may have periodic as well as life of loan caps and floors on the amount the monthly payment may be changed.

Table 1 illustrates an example of an AAM. The term of the mortgage is 30 years, carrying an interest rate of $7.0 \%$. That interest rate is based on the index rate at the time of origination $(5.0 \%)^{8}$ plus a margin over the index rate ( $2.0 \%$ ). If this were a standard fixed-rate mortgage, the borrower would make monthly, constant payments of $\$ 665.30$ throughout the entire 360 -month term of the mortgage.

With an AAM, each year, on the anniversary date of the mortgage, the current value of the index is read and the margin added to this value. Monthly payments on the AAM are then adjusted via equation 9 using the sum of the current index rate plus the margin as the interest rate. However, since the interest rate is fixed, any increase (or decrease) in the monthly payment represents an adjustment to principal amortization, not an adjustment to interest.

For example, in Table 1, the index rate rises by one percentage point by the end of the first year (month 12). As a result, the monthly payment for months $13-24$ is reset based on the $\$ 98,984.19$ remaining principal balance, 29 year ( 348 month) remaining term and an interest rate of $8.0 \%$ (current index rate plus margin). As is seen in Table 1, the payment rises to $\$ 732.43$. All of the incremental payment is credited against principal, not interest, since the interest rate is fixed.

Likewise, at the end of 24 months, the index is again observed and the payment adjusted using index plus margin as the interest rate i in equation 9.

It should also be noted that, unlike a standard fixed-rate mortgage, the term of the AAM is not affected by accelerated payments to principal. This is because the payment is adjusted annually based on retaining the original term of 30 years. ${ }^{9}$

## Valuing AAM-Backed Securities

A typical sequential payment CMO is illustrated in Tables 2 through 6. The owner of a portfolio of $\$ 420$ million (face value) 30-year term, $8 \%$ APR fixed-rate mortgages has created a 4 -tranche CMO (A,B,C and Z) - with each tranche carrying a face value of $\$ 100$ million. The residual $\$ 20$ million represents overcollateralization in the event of defaults or late payments by mortgagors.

[^4]To create claims with different maturities, the principal cash flow from the mortgage payments is sequential, i.e., all principal payments first go to Tranche A, then Tranche B, etc. Further, to create a fairly long-lived tranche, Tranche Z not only does not receive any principal payments until all the other tranches have been completely repaid, Tranche Z's coupon interest is credited as an additional principal payment against the remaining tranches, in sequence. Thus, Tranche Z receives no cash flow whatsoever until all other tranches have been completely paid off.

Assuming a positively- sloped yield curve, the required rate of return on each tranche is an increasing function of the tranche's expected duration. Hence, Tranche A, with the shortest duration, carries the lowest coupon interest rate; Tranche Z , with the longest duration, carries the highest coupon interest rate. The assumed rate of principal prepayments (in whole or in part) on the pool of underlying mortgages approximates the Public Securities Association (PSA) prepayment rate curve. ${ }^{10}$

Tables 4, 5 and 6 illustrate the sequence of principal and coupon interest payments to the various tranches as well as the expected durations and component values of the claims (interestonly and principal-only claims). Since the example assumes all tranches are sold at par, the total value of each tranche is the par value. Because Tranche Z receives no payments of any kind until all the other tranches have been extinguished, it has a substantially longer duration than tranches A, B, and C. This difference is magnified by the application of accrued interest cash flows to the other tranches.

Tables 8 through 13 illustrate the dual effects of a structural shift increase in interest rates by 2 percentage points (causing the required return on all tranches to increase by 2 percentage points) and a simultaneous reduction in the anticipated level of prepayments by one-third ( $66.667 \%$ of PSA). For every tranche, duration increases (most dramatically for tranche Z where it increases by almost one-and-a-half years) and the PO values decline significantly. For tranches A, B and C, the IO values increase marginally since the decline in prepayments results in larger and longer coupon interest payments. For tranche Z, the IO value declines since the (now more heavily discounted) coupon interest payments are further in the future.

In toto, the market value of the claims to tranches $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and Z have declined by approximately $\$ 51$ million from $\$ 400$ million to roughly $\$ 349$ million before adjusting for the effects of changes in expected duration on required returns. ${ }^{11}$

## Applying the AAM

[^5]Suppose now that the underlying mortgage pool backing the CMOs is composed of fixedrate, adjustable amortization mortgages of the type described previously. In this case, if interest rates were to rise by two percentage points, the total monthly payments on the underlying mortgage would be adjusted upward (based on a $10 \%$ interest rate rather than an $8 \%$ interest rate) with all of the additional cash inflows credited to principal.

As illustrated in Table 15, the total cash flow available for distribution to tranche holders rises significantly as the amortization is accelerated. In fact, the adjustment in the speed of amortization substantially offsets the decline in prepayments as a result of interest rates increasing. As is shown in Tables 16, 17 and 18, the expected durations of tranches A, B, C and Z return almost exactly to the expected durations of these tranches before an increase in interest rates occurred (Tables 4, 5 and 6). And the total payments to each of the tranches each period returns to roughly the same values as they were before the rise in interest rates. Thus, increasing "involuntary" payments of principal ameliorates the decline in "voluntary" prepayments.

The overall effect of the application of the AAM is best illustrated in Table 20 which summarizes the PO, IO and Total values, as well as the duration and modified duration of each tranche under all three scenarios. With the increase in interest rates, the use of the accelerated amortization mortgage increases the PO values of all four tranches significantly. Despite a decline in IO values across all four tranches (the time period over which coupon interest is paid is now shortened for all four tranches), the overall effect is to raise the cumulative market values of the four tranches by about $\$ 5$ million to $\$ 354$ million.

## Future Research

The offering of adjustable amortization mortgages as a substitute for traditional fixed-rate mortgages represents an opportunity to insulate owners of these mortgages (and the securities backed by these mortgages) from duration risk, especially in a rising interest rate environment. To more clearly identify the potential benefits, a number of issues must be addressed:

1) The relationship between required returns and expected duration. If required returns are positively correlated with expected duration, then the previous example understates the potential effect of an AAM on duration risk and mortgage-backed security prices. If securities buyers demand a premium as expected duration increases, the results will be more dramatic;
2) Would there be a secondary market for AAMs? Although the potential benefits of AAMs as opposed to standard fixed-rate mortgages could be easily articulated to investors, it is likely that AAMs would require a high level of standardization in order to be bundled together to create pools against which securities are written;
3) If there were a secondary market for AAMs, would there be a primary market? For what types of borrowers would an AAM be a suitable alternative? Since mortgage payments could rise with an AAM, some
borrowers may be reluctant to expose themselves to that risk, even though their interest rate is fixed;
4) If there were both a primary and secondary market for AAMs, to what extent should the duration risk be shared between mortgagor and mortgagee? Specifically, what would be the appropriate caps and floors on amortization adjustments - both periodically and over the life of the loan?; and,
5) Assuming a standardized AAM could be created, what is the appropriate interest rate reduction for a borrower taking out an AAM as opposed to a standard fixed-rate mortgage?

## Summary

This paper has proposed a new fixed-rate mortgage where the amortization is adjusted to reflect changes in the level of interest rates. Conceptually, the objective is to create a mortgage that has less duration risk for mortgage buyers and for buyers of securities backed by those mortgages. It is shown that such a mortgage has the potential to effectively insulate mortgagees and securities owners from changes in expected duration as a result in changes in principal prepayment patterns arising from changes in the general level of interest rates.

## Table 1

## Adjustable Amortization Mortgage Example



## Table 2

## A Typical Collateralized Mortgage Obligation (000s omitted)

Assumes Prepayment Rate of: 100\% of PSA<br>Pool Amount \$420,000<br>Interest Rate 8.000\%<br>Term 30 Years<br>Overcollateralization $\mathbf{\$ 2 0 , 0 0 0}$

| Tranche A | Required Return <br> Coupon | $7.000 \%$ <br>  <br>  <br> Amount |
| :--- | :--- | :--- |
|  |  | $\mathbf{7 . 0 0 0 \%}$ |
| Tranche B | Required Return | $7.250 \%$ |
|  | Coupon | $7.250 \%$ |
|  | Amount | $\$ 100,000$ |
| Tranche C | Required Return | $7.500 \%$ |
|  | Coupon | $7.500 \%$ |
|  | Amount | $\$ 100,000$ |
| Tranche Z | Required Return | $8.500 \%$ |
|  | Coupon | $8.500 \%$ |
|  | Amount | $\$ 100,000$ |

Notes: $\quad C M O$ is a sequential payment CMO, i.e., Tranche A receives all mortgage principal payments first, then Tranche B, then Tranche C, then Tranche Z. Tranches A, B, and C receive coupon interest beginning at the end of year 1. Tranche $Z$ receives no coupon interest until all other tranches have been completely paid off. In the interim, Tranche $Z$ is credited with accrued interest and the interest that would normally be paid to Tranche $Z$ is paid to the other tranches (in sequence) as additional principal payments.

The overcollateralization creates a residual equity class, which receives the residual cash flow each year after the claims of all tranches have been paid.

PSA stands for the Public Securities Association. A prepayment rate of " $100 \%$ of PSA" translates to a prepayment rate of approximately $2 \%$ the first year; $4 \%$ the second year; and, $6 \%$ in all subsequent years.

## Table 3

## Summary Cash Flow Information (000s omitted)

| EOY | Pool Amount | Prin \& Int Payments | Principal Prepmts | Total Amort | Interest | Amount Owed CMO Holders | tal Avail for Dist |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$420,000 |  |  |  |  | \$400,000 |  |
| 1 | 407,892 | \$37,308 | \$8,400 | \$3,708 | \$33,600 | 387,892 | \$45,708 |
| 2 | 387,653 | 36,555 | 16,316 | 3,923 | 32,631 | 367,653 | 52,870 |
| 3 | 360,328 | 35,078 | 23,259 | 4,066 | 31,012 | 340,328 | 58,338 |
| 4 | 334,583 | 32,951 | 21,620 | 4,125 | 28,826 | 314,583 | 54,571 |
| 5 | 310,324 | 30,951 | 20,075 | 4,185 | 26,767 | 290,324 | 51,026 |
| 6 | 287,459 | 29,071 | 18,619 | 4,245 | 24,826 | 267,459 | 47,690 |
| 7 | 265,906 | 27,302 | 17,248 | 4,306 | 22,997 | 245,906 | 44,550 |
| 8 | 245,585 | 25,639 | 15,954 | 4,367 | 21,273 | 225,585 | 41,594 |
| 9 | 226,422 | 24,075 | 14,735 | 4,428 | 19,647 | 206,422 | 38,810 |
| 10 | 208,346 | 22,604 | 13,585 | 4,490 | 18,114 | 188,346 | 36,189 |
| 11 | 191,292 | 21,220 | 12,501 | 4,553 | 16,668 | 171,292 | 33,721 |
| 12 | 175,199 | 19,919 | 11,478 | 4,615 | 15,303 | 155,199 | 31,396 |
| 13 | 160,009 | 18,694 | 10,512 | 4,678 | 14,016 | 140,009 | 29,206 |
| 14 | 145,668 | 17,542 | 9,601 | 4,741 | 12,801 | 125,668 | 27,142 |
| 15 | 132,124 | 16,457 | 8,740 | 4,804 | 11,653 | 112,124 | 25,197 |
| 16 | 119,330 | 15,436 | 7,927 | 4,866 | 10,570 | 99,330 | 23,363 |
| 17 | 107,243 | 14,474 | 7,160 | 4,928 | 9,546 | 87,243 | 21,634 |
| 18 | 95,819 | 13,569 | 6,435 | 4,989 | 8,579 | 75,819 | 20,003 |
| 19 | 85,021 | 12,715 | 5,749 | 5,049 | 7,666 | 65,021 | 18,464 |
| 20 | 74,812 | 11,909 | 5,101 | 5,108 | 6,802 | 54,812 | 17,011 |
| 21 | 65,159 | 11,149 | 4,489 | 5,164 | 5,985 | 45,159 | 15,638 |
| 22 | 56,031 | 10,431 | 3,910 | 5,218 | 5,213 | 36,031 | 14,340 |
| 23 | 47,402 | 9,750 | 3,362 | 5,268 | 4,483 | 27,402 | 13,112 |
| 24 | 39,245 | 9,105 | 2,844 | 5,312 | 3,792 | 19,245 | 11,949 |
| 25 | 31,541 | 8,489 | 2,355 | 5,350 | 3,140 | 11,541 | 10,844 |
| 26 | 24,272 | 7,900 | 1,892 | 5,376 | 2,523 | 4,272 | 9,792 |
| 27 | 17,429 | 7,328 | 1,456 | 5,386 | 1,942 | 0 | 8,785 |
| 28 | 11,015 | 6,763 | 1,046 | 5,369 | 1,394 | 0 | 7,809 |
| 29 | 5,058 | 6,177 | 661 | 5,296 | 881 | 0 | 6,838 |
| 30 | 0 | 5,463 | 0 | 5,058 | 405 | 0 | 5,463 |

Notes: Mortgage payments are made annually at the end of each year. Payments to tranches are made annually at the end of each year. There is no prepayment penalty on the underlying mortgages.

Table 4
Tranche A
(000s omitted)

| Coupon | $7.000 \%$ |
| :--- | :--- |
| Required Return | $7.000 \%$ |
| Duration | 2.281 years |
| Modified Duration | 2.132 |

PO Value $\quad \$ \mathbf{8 5 , 0 7 8}$
IO Value \$ 14,922
Total Value $\$ 100,000$

| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 79,392 | $\$ 20,608$ | $\$ 7,000$ | $\$ 27,608$ |
| 2 | 49,931 | 29,462 | 5,557 | 35,019 |
| 3 | 12,599 | 37,332 | 3,495 | 40,827 |
| 4 | 0 | 12,599 | 882 | 13,481 |
| 5 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| . | . | . | . | . |
| 30 | 0 | 0 | 0 | 0 |

## Tranche B <br> (000s omitted)

| Coupon | $7.250 \%$ | PO Value | $\$ 69,603$ |
| :--- | :--- | :--- | :--- |
| Required Return | $7.250 \%$ | IO Value | $\$ 30,397$ |
| Duration | 4.497 years | Total Value | $\$ 100,000$ |
| Modified Duration | 4.193 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0}$ | $\$ 100,000$ |  |  |  |
| 1 | 100,000 | $\$ 0$ | $\$ 7,250$ | $\$ 7,250$ |
| 2 | 100,000 | 0 | 7,250 | 7,250 |
| 3 | 100,000 | 0 | 7,250 | 7,250 |
| 4 | 75,998 | 24,002 | 7,250 | 31,252 |
| 5 | 39,958 | 36,039 | 5,510 | 41,549 |
| 6 | 4,313 | 35,645 | 2,897 | 38,542 |
| 7 | 0 | 4,313 | 313 | 4,625 |
| 8 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| $\cdot$ | . | . | . | . |
| 30 | 0 | 0 | 0 | 0 |

Table 5
Tranche C
(000s omitted)

| Coupon | $7.500 \%$ | PO Value | $\$ 56,067$ |
| :--- | :--- | :--- | :--- |
| Required Return | $7.500 \%$ | IO Value | $\$ 43,933$ |
| Duration | 6.297 years | Total Value | $\$ 100,000$ |
| Modified Duration | 5.858 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 100,000 | $\$ 0$ | $\$ 7,500$ | $\$ 7,500$ |
| 2 | 100,000 | 0 | 7,500 | 7,500 |
| 3 | 100,000 | 0 | 7,500 | 7,500 |
| 4 | 100,000 | 0 | 7,500 | 7,500 |
| 5 | 100,000 | 0 | 7,500 | 7,500 |
| 6 | 100,000 | 0 | 7,500 | 7,500 |
| 7 | 68,892 | 31,108 | 7,500 | 38,608 |
| 8 | 33,525 | 35,367 | 5,167 | 40,534 |
| 9 | 0 | 33,525 | 2,514 | 36,039 |
| 10 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| . | 0 | . | . | . |
| 30 | 0 | 0 | 0 | . |

## Table 6

Tranche Z (000s omitted)

| Coupon | $8.500 \%$ | PO Value | $\$ 49,405$ |
| :--- | :--- | :--- | :--- |
| Required Return | $8.500 \%$ | IO Value | $\$ 50,595$ |
| Duration | 14.458 years | Total Value | $\$ 100,000$ |
| Modified Duration | 13.325 years |  |  |


| EOY | Amount Owed | Accrued Interest | Accum Accrued Int | Principal Payments | Interest Payments | Total Payments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$100,000 |  |  |  |  |  |
| 1 | 108,500 | \$8,500 | \$8,500 | \$0 | \$0 | \$0 |
| 2 | 117,723 | 9,223 | 17,723 | 0 | 0 | 0 |
| 3 | 127,729 | 10,006 | 27,729 | 0 | 0 | 0 |
| 4 | 138,586 | 10,857 | 38,586 | 0 | 0 | 0 |
| 5 | 150,366 | 11,780 | 50,366 | 0 | 0 | 0 |
| 6 | 163,147 | 12,781 | 63,147 | 0 | 0 | 0 |
| 7 | 177,014 | 13,867 | 77,014 | 0 | 0 | 0 |
| 8 | 192,060 | 15,046 | 92,060 | 0 | 0 | 0 |
| 9 | 206,422 | 14,361 | 106,422 | $(14,361)$ | 16,325 | 1,964 |
| 10 | 188,346 | $(18,076)$ | 88,346 | 18,076 | 17,546 | 35,622 |
| 11 | 171,292 | $(17,054)$ | 71,292 | 17,054 | 16,009 | 33,063 |
| 12 | 155,199 | $(16,093)$ | 55,199 | 16,093 | 14,560 | 30,653 |
| 13 | 140,009 | $(15,190)$ | 40,009 | 15,190 | 13,192 | 28,382 |
| 14 | 125,668 | $(14,342)$ | 25,668 | 14,342 | 11,901 | 26,242 |
| 15 | 112,124 | $(13,544)$ | 12,124 | 13,544 | 10,682 | 24,225 |
| 16 | 99,330 | $(12,794)$ | (670) | 12,794 | 9,531 | 22,324 |
| 17 | 87,243 | $(12,088)$ | $(12,757)$ | 12,088 | 8,443 | 20,531 |
| 18 | 75,819 | $(11,424)$ | $(24,181)$ | 11,424 | 7,416 | 18,839 |
| 19 | 65,021 | $(10,798)$ | $(34,979)$ | 10,798 | 6,445 | 17,243 |
| 20 | 54,812 | $(10,209)$ | $(45,188)$ | 10,209 | 5,527 | 15,736 |
| 21 | 45,159 | $(9,653)$ | $(54,841)$ | 9,653 | 4,659 | 14,312 |
| 22 | 36,031 | $(9,127)$ | $(63,969)$ | 9,127 | 3,838 | 12,966 |
| 23 | 27,402 | $(8,630)$ | $(72,598)$ | 8,630 | 3,063 | 11,692 |
| 24 | 19,245 | $(8,157)$ | $(80,755)$ | 8,157 | 2,329 | 10,486 |
| 25 | 11,541 | $(7,704)$ | $(88,459)$ | 7,704 | 1,636 | 9,340 |
| 26 | 4,272 | $(7,269)$ | $(95,728)$ | 7,269 | 981 | 8,250 |
| 27 | 0 | $(4,272)$ | $(100,000)$ | 4,272 | 363 | 4,635 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |

## Table 7

Residual Equity Class
(000s omitted)
Prepayment $=\quad 100.000 \%$ of PSA
IRR to Residual Equity Class
8.621\%

| EOY | Total Cash Flow to Pool | Total Payments to $A, B, C$ \& Z Tranches | Residual Cash Flow to Equity |
| :---: | :---: | :---: | :---: |
| 0 |  |  | (\$20,000) |
| 1 | \$45,708 | \$42,358 | 3,350 |
| 2 | 52,870 | 49,769 | 3,101 |
| 3 | 58,338 | 55,577 | 2,761 |
| 4 | 54,571 | 52,234 | 2,337 |
| 5 | 51,026 | 49,049 | 1,977 |
| 6 | 47,690 | 46,042 | 1,648 |
| 7 | 44,550 | 43,233 | 1,317 |
| 8 | 41,594 | 40,534 | 1,059 |
| 9 | 38,810 | 38,003 | 807 |
| 10 | 36,189 | 35,622 | 568 |
| 11 | 33,721 | 33,063 | 658 |
| 12 | 31,396 | 30,653 | 744 |
| 13 | 29,206 | 28,382 | 824 |
| 14 | 27,142 | 26,242 | 900 |
| 15 | 25,197 | 24,225 | 972 |
| 16 | 23,363 | 22,324 | 1,039 |
| 17 | 21,634 | 20,531 | 1,103 |
| 18 | 20,003 | 18,839 | 1,164 |
| 19 | 18,464 | 17,243 | 1,221 |
| 20 | 17,011 | 15,736 | 1,275 |
| 21 | 15,638 | 14,312 | 1,326 |
| 22 | 14,340 | 12,966 | 1,374 |
| 23 | 13,112 | 11,692 | 1,420 |
| 24 | 11,949 | 10,486 | 1,463 |
| 25 | 10,844 | 9,340 | 1,504 |
| 26 | 9,792 | 8,250 | 1,542 |
| 27 | 8,785 | 4,635 | 4,149 |
| 28 | 7,809 | 0 | 7,809 |
| 29 | 6,838 | 0 | 6,838 |
| 30 | 5,463 | 0 | 5,463 |

## Table 8



## Table 9

Summary Cash Flow Information (000s omitted)

| EOY | Pool Amount | Prin \& Int Payments | Principal Prepmts | Total Amort | Interest | Amt Owed CMO Holders | tal Avail for Dist |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$420,000 |  |  |  |  | \$400,000 |  |
| 1 | 410,692 | \$37,308 | \$5,600 | \$3,708 | \$33,600 | 390,692 | \$42,908 |
| 2 | 395,790 | 36,806 | 10,952 | 3,950 | 32,855 | 375,790 | 47,757 |
| 3 | 375,807 | 35,815 | 15,832 | 4,151 | 31,663 | 355,807 | 51,646 |
| 4 | 356,473 | 34,367 | 15,032 | 4,302 | 30,065 | 336,473 | 49,399 |
| 5 | 337,755 | 32,976 | 14,259 | 4,458 | 28,518 | 317,755 | 47,235 |
| 6 | 319,625 | 31,641 | 13,510 | 4,620 | 27,020 | 299,625 | 45,151 |
| 7 | 302,053 | 30,357 | 12,785 | 4,787 | 25,570 | 282,053 | 43,142 |
| 8 | 285,010 | 29,125 | 12,082 | 4,960 | 24,164 | 265,010 | 41,207 |
| 9 | 268,471 | 27,940 | 11,400 | 5,139 | 22,801 | 248,471 | 39,341 |
| 10 | 252,407 | 26,802 | 10,739 | 5,324 | 21,478 | 232,407 | 37,541 |
| 11 | 236,795 | 25,708 | 10,096 | 5,516 | 20,193 | 216,795 | 35,805 |
| 12 | 221,610 | 24,657 | 9,472 | 5,713 | 18,944 | 201,610 | 34,129 |
| 13 | 206,828 | 23,646 | 8,864 | 5,917 | 17,729 | 186,828 | 32,511 |
| 14 | 192,427 | 22,674 | 8,273 | 6,128 | 16,546 | 172,427 | 30,948 |
| 15 | 178,384 | 21,740 | 7,697 | 6,346 | 15,394 | 158,384 | 29,437 |
| 16 | 164,679 | 20,841 | 7,135 | 6,570 | 14,271 | 144,679 | 27,976 |
| 17 | 151,291 | 19,975 | 6,587 | 6,801 | 13,174 | 131,291 | 26,562 |
| 18 | 138,201 | 19,142 | 6,052 | 7,038 | 12,103 | 118,201 | 25,193 |
| 19 | 125,391 | 18,339 | 5,528 | 7,283 | 11,056 | 105,391 | 23,867 |
| 20 | 112,842 | 17,564 | 5,016 | 7,533 | 10,031 | 92,842 | 22,580 |
| 21 | 100,539 | 16,817 | 4,514 | 7,789 | 9,027 | 80,539 | 21,330 |
| 22 | 88,466 | 16,094 | 4,022 | 8,051 | 8,043 | 68,466 | 20,116 |
| 23 | 76,610 | 15,394 | 3,539 | 8,317 | 7,077 | 56,610 | 18,933 |
| 24 | 64,960 | 14,715 | 3,064 | 8,586 | 6,129 | 44,960 | 17,779 |
| 25 | 53,507 | 14,052 | 2,598 | 8,855 | 5,197 | 33,507 | 16,650 |
| 26 | 42,246 | 13,401 | 2,140 | 9,121 | 4,281 | 22,246 | 15,541 |
| 27 | 31,181 | 12,755 | 1,690 | 9,375 | 3,380 | 11,181 | 14,445 |
| 28 | 20,329 | 12,099 | 1,247 | 9,605 | 2,494 | 329 | 13,346 |
| 29 | 9,742 | 11,400 | 813 | 9,773 | 1,626 | 0 | 12,213 |
| 30 | 0 | 10,522 | 0 | 9,742 | 779 | 0 | 10,522 |

Table 10

## Tranche A

(000s omitted)

| Coupon | $7.000 \%$ | PO Value | $\$ 79,691$ |
| :--- | :--- | :--- | :--- |
| Required Return | $9.000 \%$ | IO Value | $\$ 15,796$ |
| Duration | 2.483 years | Total Value | $\$ 95,487$ |
| Modified Duration | 2.278 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 82,192 | $\$ 17,808$ | $\$ 7,000$ | $\$ 24,808$ |
| 2 | 58,068 | 24,125 | 5,753 | 29,878 |
| 3 | 28,078 | 29,989 | 4,065 | 34,054 |
| 4 | 0 | 28,078 | 1,965 | 30,044 |
| 5 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| . | . | . | . | . |
| 30 | 0 | 0 | 0 | 0 |

Tranche B (000s omitted)

| Coupon | $7.250 \%$ | PO Value | $\$ 58,666$ |
| :--- | :--- | :--- | :--- |
| Required Return | $9.250 \%$ | IO Value | $\$ 32,397$ |
| Duration | 5.036 years | Total Value | $\$ 91,063$ |
| Modified Duration | 4.610 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 100,000 | $\$ 0$ | $\$ 7,250$ | $\$ 7,250$ |
| 2 | 100,000 | 0 | 7,250 | 7,250 |
| 3 | 100,000 | 0 | 7,250 | 7,250 |
| 4 | 97,887 | 2,113 | 7,250 | 9,363 |
| 5 | 67,390 | 30,497 | 7,097 | 37,594 |
| 6 | 36,478 | 30,911 | 4,886 | 35,797 |
| 7 | 5,039 | 31,440 | 2,645 | 34,084 |
| 8 | 0 | 5,039 | 365 | 5,404 |
| 9 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| . | 0 | 0 | 0 | 0 |

Table 11

## Tranche C <br> (000s omitted)

| Coupon | $7.500 \%$ | PO Value | $\$ 43,563$ |
| :--- | :--- | :--- | :--- |
| Required Return | $9.500 \%$ | IO Value | $\$ 44,556$ |
| Duration | 6.777 years | Total Value | $\$ 88,119$ |
| Modified Duration | 6.189 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 100,000 | $\$ 0$ | $\$ 7,500$ | $\$ 7,500$ |
| 2 | 100,000 | 0 | 7,500 | 7,500 |
| 3 | 100,000 | 0 | 7,500 | 7,500 |
| 4 | 100,000 | 0 | 7,500 | 7,500 |
| 5 | 100,000 | 0 | 7,500 | 7,500 |
| 6 | 100,000 | 0 | 7,500 | 7,500 |
| 7 | 100,000 | 0 | 7,500 | 7,500 |
| 8 | 72,950 | 27,050 | 7,500 | 34,550 |
| 9 | 40,085 | 32,865 | 5,471 | 38,336 |
| 10 | 6,309 | 33,776 | 3,006 | 36,782 |
| 11 | 0 | 6,309 | 473 | 6,782 |
| 12 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| . | . | . | . | . |
| . | 0 | 0 | 0 | 0 |

Table 12

## Tranche Z <br> (000s omitted)

| Coupon | $8.500 \%$ | PO Value | $\$ 37,760$ |
| :--- | :--- | :--- | :--- |
| Required Return | $10.500 \%$ | IO Value | $\$ 36,870$ |
| Duration | 15.851 years | Total Value | $\$ 74,630$ |
| Modified Duration | 14.345 years |  |  |


| EOY | Amount Owed | Accrued Interest | Accum Accrued Int | Principal Payments | Interest Payments | Total Payments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$100,000 |  |  |  |  |  |
| 1 | 108,500 | \$8,500 | \$8,500 | \$0 | \$0 | \$0 |
| 2 | 117,723 | 9,223 | 17,723 | 0 | 0 | 0 |
| 3 | 127,729 | 10,006 | 27,729 | 0 | 0 | 0 |
| 4 | 138,586 | 10,857 | 38,586 | 0 | 0 | 0 |
| 5 | 150,366 | 11,780 | 50,366 | 0 | 0 | 0 |
| 6 | 163,147 | 12,781 | 63,147 | 0 | 0 | 0 |
| 7 | 177,014 | 13,867 | 77,014 | 0 | 0 | 0 |
| 8 | 192,060 | 15,046 | 92,060 | 0 | 0 | 0 |
| 9 | 208,386 | 16,325 | 108,386 | 0 | 0 | 0 |
| 10 | 226,098 | 17,713 | 126,098 | 0 | 0 | 0 |
| 11 | 216,795 | $(9,303)$ | 116,795 | 9,303 | 19,218 | 28,521 |
| 12 | 201,610 | $(15,185)$ | 101,610 | 15,185 | 18,428 | 33,613 |
| 13 | 186,828 | $(14,782)$ | 86,828 | 14,782 | 17,137 | 31,919 |
| 14 | 172,427 | $(14,401)$ | 72,427 | 14,401 | 15,880 | 30,282 |
| 15 | 158,384 | $(14,043)$ | 58,384 | 14,043 | 14,656 | 28,699 |
| 16 | 144,679 | $(13,705)$ | 44,679 | 13,705 | 13,463 | 27,168 |
| 17 | 131,291 | $(13,388)$ | 31,291 | 13,388 | 12,298 | 25,686 |
| 18 | 118,201 | $(13,090)$ | 18,201 | 13,090 | 11,160 | 24,250 |
| 19 | 105,391 | $(12,811)$ | 5,391 | 12,811 | 10,047 | 22,858 |
| 20 | 92,842 | $(12,549)$ | $(7,158)$ | 12,549 | 8,958 | 21,507 |
| 21 | 80,539 | $(12,303)$ | $(19,461)$ | 12,303 | 7,892 | 20,195 |
| 22 | 68,466 | $(12,073)$ | $(31,534)$ | 12,073 | 6,846 | 18,919 |
| 23 | 56,610 | $(11,856)$ | $(43,390)$ | 11,856 | 5,820 | 17,675 |
| 24 | 44,960 | $(11,650)$ | $(55,040)$ | 11,650 | 4,812 | 16,462 |
| 25 | 33,507 | $(11,453)$ | $(66,493)$ | 11,453 | 3,822 | 15,275 |
| 26 | 22,246 | $(11,261)$ | $(77,754)$ | 11,261 | 2,848 | 14,109 |
| 27 | 11,181 | $(11,065)$ | $(88,819)$ | 11,065 | 1,891 | 12,956 |
| 28 | 329 | $(10,852)$ | $(99,671)$ | 10,852 | 950 | 11,802 |
| 29 | 0 | (329) | $(100,000)$ | 329 | 28 | 357 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |

## Table 13

## Residual Equity Class

(000s omitted)

| Prepayment | $=$ | $66.667 \%$ of PSA |
| ---: | :--- | :--- |
| IRR to Residual Equity Class |  | $8.662 \%$ |

\(\left.$$
\begin{array}{rrrr}\text { EOY } & \begin{array}{r}\text { Total } \\
\text { Cash Flow } \\
\text { to Pool }\end{array} & \begin{array}{r}\text { Total Payments } \\
\text { to A,B,C \& Z } \\
\text { Tranches }\end{array} & \begin{array}{r}\text { Residual } \\
\text { Cash Flow } \\
\text { to Equity }\end{array} \\
\hline 0 & & & \begin{array}{r}(\$ 20,000) \\
1\end{array}
$$ <br>

2 \& \$ 42,908 \& \$ 39,558 \& 3,350\end{array}\right]\)| 3,129 |
| :--- |
| 3 |

## Table 14

Interest Rates Rise by 2.0\%; Principal Amortization Accelerated
Assumes Prepayment Rate of: $\mathbf{6 6 . 6 6 7 \%}$ of PSA
Pool Amount \$420,000
Interest Rate 8.000\%
Term 30 Years
Overcollateralization \$20,000

| Tranche A | Required Return Coupon Amount | $\begin{aligned} & 9.000 \% \\ & 7.000 \% \\ & \$ 100,000 \end{aligned}$ |
| :---: | :---: | :---: |
| Tranche B | Required Return | 9.250\% |
|  | Coupon | 7.250\% |
|  | Amount | \$100,000 |
| Tranche C | Required Return | 9.500\% |
|  | Coupon | 7.500\% |
|  | Amount | \$100,000 |
| Tranche Z | Required Return | 10.500\% |
|  | Coupon | 8.500\% |
|  | Amount | \$100,000 |

## Table 15

## Summary Cash Flow Information (000s omitted)

| EOY | Pool Amount | Prin \& Int Payments | Principal Prepmts | Total Amort | Interest | Amt Owed CMO Holders | tal Avail for Dist |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$420,000 |  |  |  |  | \$400,000 |  |
| 1 | 403,447 | \$44,553 | \$5,600 | \$10,953 | \$33,600 | 383,447 | \$50,153 |
| 2 | 381,905 | 43,059 | 10,759 | 10,783 | 32,276 | 361,905 | 53,818 |
| 3 | 356,145 | 41,036 | 15,276 | 10,484 | 30,552 | 336,145 | 56,312 |
| 4 | 331,835 | 38,555 | 14,246 | 10,064 | 28,492 | 311,835 | 52,801 |
| 5 | 308,886 | 36,223 | 13,273 | 9,676 | 26,547 | 288,886 | 49,496 |
| 6 | 287,212 | 34,029 | 12,355 | 9,318 | 24,711 | 267,212 | 46,385 |
| 7 | 266,734 | 31,967 | 11,488 | 8,990 | 22,977 | 246,734 | 43,455 |
| 8 | 247,376 | 30,027 | 10,669 | 8,688 | 21,339 | 227,376 | 40,696 |
| 9 | 229,069 | 28,202 | 9,895 | 8,412 | 19,790 | 209,069 | 38,097 |
| 10 | 211,746 | 26,486 | 9,163 | 8,160 | 18,326 | 191,746 | 35,649 |
| 11 | 195,344 | 24,872 | 8,470 | 7,932 | 16,940 | 175,344 | 33,341 |
| 12 | 179,805 | 23,353 | 7,814 | 7,725 | 15,628 | 159,805 | 31,167 |
| 13 | 165,074 | 21,924 | 7,192 | 7,539 | 14,384 | 145,074 | 29,116 |
| 14 | 151,098 | 20,579 | 6,603 | 7,373 | 13,206 | 131,098 | 27,182 |
| 15 | 137,829 | 19,313 | 6,044 | 7,225 | 12,088 | 117,829 | 25,357 |
| 16 | 125,221 | 18,121 | 5,513 | 7,095 | 11,026 | 105,221 | 23,634 |
| 17 | 113,232 | 16,998 | 5,009 | 6,981 | 10,018 | 93,232 | 22,007 |
| 18 | 101,821 | 15,941 | 4,529 | 6,882 | 9,059 | 81,821 | 20,470 |
| 19 | 90,950 | 14,944 | 4,073 | 6,798 | 8,146 | 70,950 | 19,016 |
| 20 | 80,585 | 14,003 | 3,638 | 6,727 | 7,276 | 60,585 | 17,641 |
| 21 | 70,693 | 13,115 | 3,223 | 6,668 | 6,447 | 50,693 | 16,338 |
| 22 | 61,246 | 12,275 | 2,828 | 6,620 | 5,655 | 41,246 | 15,103 |
| 23 | 52,216 | 11,480 | 2,450 | 6,581 | 4,900 | 32,216 | 13,930 |
| 24 | 43,579 | 10,725 | 2,089 | 6,548 | 4,177 | 23,579 | 12,814 |
| 25 | 35,316 | 10,006 | 1,743 | 6,520 | 3,486 | 15,316 | 11,749 |
| 26 | 27,412 | 9,316 | 1,413 | 6,491 | 2,825 | 7,412 | 10,729 |
| 27 | 19,861 | 8,648 | 1,096 | 6,455 | 2,193 | 0 | 9,744 |
| 28 | 12,669 | 7,986 | 794 | 6,398 | 1,589 | 0 | 8,781 |
| 29 | 5,876 | 7,300 | 507 | 6,286 | 1,014 | 0 | 7,807 |
| 30 | 0 | 6,346 | 0 | 5,876 | 470 | 0 | 6,346 |

Table 16

## Tranche A <br> (000s omitted)

| Coupon | $7.000 \%$ | PO Value | $\$ 82,459$ |
| :--- | :--- | :--- | :--- |
| Required Return | $9.000 \%$ | IO Value | $\$ 13,643$ |
| Duration | 2.139 years | Total Value | $\$ 96,102$ |
| Modified Duration | 1.962 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 74,947 | $\$ 25,053$ | $\$ 7,000$ | $\$ 32,053$ |
| 2 | 44,182 | 30,764 | 5,246 | 36,011 |
| 3 | 8,416 | 35,766 | 3,093 | 38,859 |
| 4 | 0 | 8,416 | 589 | 9,005 |
| 5 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| . | . | . | . | . |
| . | 0 | 0 | 0 | 0 |

Tranche B
(000s omitted)

| Coupon | $7.250 \%$ | PO Value | $\$ 63,545$ |
| :--- | :--- | :--- | :--- |
| Required Return | $9.250 \%$ | IO Value | $\$ 28,573$ |
| Duration | 4.423 years | Total Value | $\$ 92,128$ |
| Modified Duration | 4.049 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0}$ | $\$ 100,000$ |  |  |  |
| 1 | 100,000 | $\$ 0$ | $\$ 7,250$ | $\$ 7,250$ |
| 2 | 100,000 | 0 | 7,250 | 7,250 |
| 3 | 100,000 | 0 | 7,250 | 7,250 |
| 4 | 73,249 | 26,751 | 7,250 | 34,001 |
| 5 | 38,520 | 34,729 | 5,311 | 40,040 |
| 6 | 4,065 | 34,455 | 2,793 | 37,248 |
| 7 | 0 | 4,065 | 295 | 4,360 |
| 8 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| 30 | 0 | 0 | 0 | 0 |

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Table 17
Tranche C
(000s omitted)

| Coupon | $7.500 \%$ | PO Value | $\$ 48,266$ |
| :--- | :--- | :--- | :--- |
| Required Return | $9.500 \%$ | IO Value | $\$ 40,843$ |
| Duration | 6.195 years | Total Value | $\$ 89,109$ |
| Modified Duration | 5.568 years |  |  |


| EOY | Amount <br> Owed | Principal <br> Allocation | Coupon <br> Interest | Total <br> Payments |
| ---: | ---: | ---: | ---: | ---: |
| 0 | $\$ 100,000$ |  |  |  |
| 1 | 100,000 | $\$ 0$ | $\$ 7,500$ | $\$ 7,500$ |
| 2 | 100,000 | 0 | 7,500 | 7,500 |
| 3 | 100,000 | 0 | 7,500 | 7,500 |
| 4 | 100,000 | 0 | 7,500 | 7,500 |
| 5 | 100,000 | 0 | 7,500 | 7,500 |
| 6 | 100,000 | 0 | 7,500 | 7,500 |
| 7 | 69,720 | 30,280 | 7,500 | 37,780 |
| 8 | 35,316 | 34,404 | 5,229 | 39,633 |
| 9 | 684 | 34,632 | 2,649 | 37,281 |
| 10 | 0 | 684 | 51 | 735 |
| 11 | 0 | 0 | 0 | 0 |
| . | . | . | . | . |
| $\cdot$ | . | . | . | . |
|  | 0 | 0 | 0 | 0 |

Table 18

## Tranche Z <br> (000s omitted)

| Coupon | $8.500 \%$ | PO Value | $\$ 42,141$ |
| :--- | :--- | :--- | :--- |
| Required Return | $10.500 \%$ | IO Value | $\$ 34,567$ |
| Duration | 14.361 years | Total Value | $\$ 76,708$ |
| Modified Duration | 12.996 years |  |  |


| EOY | Amount Owed | Accrued Interest | Accum Accrued Int | Principal Payments | Interest Payments | Total Payments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$100,000 |  |  |  |  |  |
| 1 | 108,500 | \$8,500 | \$8,500 | \$0 | \$0 | \$0 |
| 2 | 117,723 | 9,223 | 17,723 | 0 | 0 | 0 |
| 3 | 127,729 | 10,006 | 27,729 | 0 | 0 | 0 |
| 4 | 138,586 | 10,857 | 38,586 | 0 | 0 | 0 |
| 5 | 150,366 | 11,780 | 50,366 | 0 | 0 | 0 |
| 6 | 163,147 | 12,781 | 63,147 | 0 | 0 | 0 |
| 7 | 177,014 | 13,867 | 77,014 | 0 | 0 | 0 |
| 8 | 192,060 | 15,046 | 92,060 | 0 | 0 | 0 |
| 9 | 208,386 | 16,325 | 108,386 | 0 | 0 | 0 |
| 10 | 191,746 | $(16,639)$ | 91,746 | 16,639 | 17,713 | 34,352 |
| 11 | 175,344 | $(16,402)$ | 75,344 | 16,402 | 16,298 | 32,700 |
| 12 | 159,805 | $(15,539)$ | 59,805 | 15,539 | 14,904 | 30,443 |
| 13 | 145,074 | $(14,731)$ | 45,074 | 14,731 | 13,583 | 28,315 |
| 14 | 131,098 | $(13,976)$ | 31,098 | 13,976 | 12,331 | 26,307 |
| 15 | 117,829 | $(13,269)$ | 17,829 | 13,269 | 11,143 | 24,412 |
| 16 | 105,221 | $(12,608)$ | 5,221 | 12,608 | 10,015 | 22,623 |
| 17 | 93,232 | $(11,989)$ | $(6,768)$ | 11,989 | 8,944 | 20,933 |
| 18 | 81,821 | $(11,411)$ | $(18,179)$ | 11,411 | 7,925 | 19,336 |
| 19 | 70,950 | $(10,871)$ | $(29,050)$ | 10,871 | 6,955 | 17,825 |
| 20 | 60,585 | $(10,365)$ | $(39,415)$ | 10,365 | 6,031 | 16,396 |
| 21 | 50,693 | $(9,891)$ | $(49,307)$ | 9,891 | 5,150 | 15,041 |
| 22 | 41,246 | $(9,448)$ | $(58,754)$ | 9,448 | 4,309 | 13,756 |
| 23 | 32,216 | $(9,030)$ | $(67,784)$ | 9,030 | 3,506 | 12,536 |
| 24 | 23,579 | $(8,637)$ | $(76,421)$ | 8,637 | 2,738 | 11,375 |
| 25 | 15,316 | $(8,263)$ | $(84,684)$ | 8,263 | 2,004 | 10,267 |
| 26 | 7,412 | $(7,904)$ | $(92,588)$ | 7,904 | 1,302 | 9,205 |
| 27 | 0 | $(7,412)$ | $(100,000)$ | 7,412 | 630 | 8,042 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 19

## Residual Equity Class <br> (000s omitted)

Prepayment =
IRR to Residual Equity Class
66.667\% of PSA
8.488\%

| EOY |  | Total Payments to $A, B, C$ \& $Z$ Tranches | Residual Cash Flow to Equity |
| :---: | :---: | :---: | :---: |
| 0 |  |  | $(\$ 20,000)$ |
| 1 | \$50,153 | \$46,803 | 3,350 |
| 2 | 53,818 | 50,761 | 3,057 |
| 3 | 56,312 | 53,609 | 2,703 |
| 4 | 52,801 | 50,506 | 2,296 |
| 5 | 49,496 | 47,540 | 1,956 |
| 6 | 46,385 | 44,748 | 1,637 |
| 7 | 43,455 | 42,140 | 1,315 |
| 8 | 40,696 | 39,633 | 1,064 |
| 9 | 38,097 | 37,281 | 816 |
| 10 | 35,649 | 35,087 | 561 |
| 11 | 33,341 | 32,700 | 641 |
| 12 | 31,167 | 30,443 | 723 |
| 13 | 29,116 | 28,315 | 801 |
| 14 | 27,182 | 26,307 | 875 |
| 15 | 25,357 | 24,412 | 945 |
| 16 | 23,634 | 22,623 | 1,011 |
| 17 | 22,007 | 20,933 | 1,074 |
| 18 | 20,470 | 19,336 | 1,134 |
| 19 | 19,016 | 17,825 | 1,191 |
| 20 | 17,641 | 16,396 | 1,245 |
| 21 | 16,338 | 15,041 | 1,297 |
| 22 | 15,103 | 13,756 | 1,347 |
| 23 | 13,930 | 12,536 | 1,394 |
| 24 | 12,814 | 11,375 | 1,439 |
| 25 | 11,749 | 10,267 | 1,482 |
| 26 | 10,729 | 9,205 | 1,523 |
| 27 | 9,744 | 8,042 | 1,702 |
| 28 | 8,781 | 0 | 8,781 |
| 29 | 7,807 | 0 | 7,807 |
| 30 | 6,346 | 0 | 6,346 |

## Table 20

## Summary Statistics

| Tranche |  | Scenario |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Original | Interest Rates Rise by 2\% |  |
|  |  | FRM | AAM |
| A | PO Value |  | \$ 85,078 | \$ 79,691 | \$ 82,459 |
|  | IO Value | 14,922 | 15,796 | 13,643 |
|  | Total Value | 100,000 | 95,487 | 96,102 |
|  | Duration | 2.281 | 2.483 | 2.139 |
|  | Modified Duration | 2.132 | 2.278 | 1.962 |
| B | PO Value | 69,603 | 58,666 | 63,545 |
|  | IO Value | 30,397 | 32,397 | 28,573 |
|  | Total Value | 100,000 | 91,063 | 92,128 |
|  | Duration | 4.497 | 5.036 | 4.423 |
|  | Modified Duration | 4.193 | 4.610 | 4.049 |
| C | PO Value | 56,067 | 43,563 | 48,266 |
|  | IO Value | 43,933 | 44,556 | 40,843 |
|  | Total Value | 100,000 | 88,119 | 89,109 |
|  | Duration | 6.297 | 6.777 | 6.195 |
|  | Modified Duration | 5.858 | 6.189 | 5.568 |
| Z | PO Value | 49,405 | 37,760 | 42,141 |
|  | IO Value | 50,595 | 36,870 | 34,567 |
|  | Total Value | 100,000 | 74,630 | 76,708 |
|  | Duration | 14.458 | 15.851 | 14.361 |
|  | Modified Duration | 13.325 | 14.345 | 12.996 |

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[^0]:    ${ }^{1}$ Note that the summation in the denominator on the right-hand side of equation 1 is the PRICE of the security.
    ${ }^{2}$ As in the case of a fixed coupon, non-callable, default-free security such as a US government bond. Where cash flows each period may change as yields change (e.g., callable debt, floating rate debt, or where principal prepayments may also change), it is more appropriate to place an expectations operator ( E ) in front of the periodic cash flows variable $\left(\mathrm{CF}_{\mathrm{t}}\right)$ in equation 1.

[^1]:    ${ }^{3}$ To broaden the duration across tranches, so-called "companion" CMOs permit the longest maturity tranche to accrue interest as long as any other tranche is still "alive." This accrued interest is paid in cash to earlier-maturing tranches as a principal allocation, reducing their duration. The accrued interest is credited to the later-maturing tranche as marked-up principal.

[^2]:    4 Refinancings are dampened by what is known as "burnout." Even though it may be financially attractive to refinance, some mortgagors do not refinance for a variety of reasons: inability to qualify for a new mortgage; ignorance of the benefits of refinancing; transactions costs, etc.

    5 Foreclosures also tend to increase in a rising interest rate environment that may accelerate prepayments and dampen price declines. However, foreclosures may not permit mortgagors and security holders to recover the entire principal value of their investments.

[^3]:    ${ }^{6}$ If prepayments decline, then principal payments to all tranches occur over a longer period of time. As a result, coupon interest payments continue for a longer period of time. Similarly, if prepayments increase, principal payments to all tranches occur over a shorter period of time. Thus, coupon interest payments are made over a shorter time period.
    ${ }^{7}$ Similarly, amortization is decelerated in response to a downward change in interest rates. However, as mortgage interest rates decline, home sales and refinancings will accelerate, mitigating the deceleration in amortization.

[^4]:    ${ }^{8}$ Analogous to an ARM, the underlying index could be a US Treasury security rate, for example, the rate on 10year US Treasury bonds.
    ${ }^{9}$ As a result, the AAM may provide some modest duration risk protection for the mortgagee as interest rates fall since the mortgagor's monthly payment also declines. Some mortgagors, who might otherwise refinance as interest rates fall, may elect, instead, to continue making the (now lower) monthly payments on the AAM. This is aptly illustrated in Table 1 beginning in month 49. The index rate plus margin has returned to its original level (7.0\%). However, because principal payments were accelerated in months 13 through 48, the total monthly payment beginning in month 49 is lower than the original total monthly payment (\$642.93 v. \$665.30).

[^5]:    ${ }^{10}$ The PSA prepayment rate curve (" $100 \%$ of PSA") assumes prepayments of 0.2 percent per month the first year, then increases by 0.2 percent per month through month 30 , then levels off at 0.5 percent per month ( 6 percent annually) for the remaining stated maturity. See, for example, Brueggeman and Fisher (1997). Because the example used here assumes annual payments, the assumed prepayment rate that corresponds to " $100 \%$ of PSA" is 2 percent the first year, 4 percent the second year, and 6 percent in all subsequent years.
    ${ }^{11}$ The reported market values for tranches A, B, C, and Z in Tables 10 through 12 overstate the "true" market values if required returns are positively correlated with expected duration.

