BUBBLE GEOMETRY AND CHAOTIC PRICING BEHAVIOUR*

Christopher Wright[1], **Christopher Eves**[2], and **Rodney Jefferies**[3]

[1]Professor of Accounting, and Director of CAER,
[2]Professor of Property Studies,
[3]Associate Professor in Property Studies
Commerce Division, Lincoln University, New Zealand

Corresponding author: wrightc@lincoln.ac.nz

Abstract

This paper is a deductive theoretical enquiry into the flow of effects from the geometry of price bubbles/busts, to price indices, to pricing behaviours of sellers and buyers, and back to price bubbles/busts. The intent of the analysis is to suggest analytical approaches to identify the presence, maturity, and/or sustainability of a price bubble. We present a pricing model to emulate market behaviour, including numeric examples and charts of the interaction of supply and demand. The model extends into dynamic market solutions myopic (single- and multi-period) backward looking rational expectations to demonstrate how buyers and sellers interact to affect supply and demand and to show how capital gain expectations can be a destabilising influence – i.e. the lagged effects of past price gains can drive the market price away from long-run market-worth. Investing based on the outputs of past price-based valuation models appear to be more of a game-of-chance than a sound investment strategy.

Keywords: Fisher, market behaviour, market equilibrium, market-worth, price, price bubble, rational expectations, value-estimate.

Introduction

Buy cheap and sell dear is sound, ubiquitous trading advice that traces from Warren Buffet's (2000) strategy on technical analysis, back through Hetty Green (1834-1916) as *The Witch of Wall Street* (Garrison, 1982), to *A New View of Society* by Robert Owen (1816), to Adam Smith's *Wealth of Nations* (Bk 4, Ch 2, 1776), to St. Augustine (On the Holy Trinity, Bk XIII, Ch 4, 400) – who attributed it even further back – to the Poet Ennius (239-169 BCE) of the early Roman Republic.[1] An earlier writing by Aristotle (350 BCE) attributes the concept of *buy low and sell high* to the philosopher *Thales the Milesian* (6th Century BCE). This ancient, time-tested maxim epitomises this paper's first three theorems:

1) Price differs from value—even if drawn from a free-market, arms-length sale, price may be: Greater than, Equal to, or Less than Value.

A Price-Value axiom and corollary supports the first theorem by drawing from rational economic behaviour to show that a seller/(buyer) will only freely sell/(buy) if the:

1a) Sales price (P_s) is above the value a seller places on what s/he is selling, and

1b) Sales price (P_s) is below the value a buyer places on what s/he is buying.

***** Derived from Chaos Theory, described by James P. Crutchfield of the Santa Cruz *Dynamical Systems Collective*, in the late 1970's, as *'… behavior that produces information (amplifies small uncertainties), but is not utterly unpredictable'* (Gleick, 1988, p.306)

[1] St. Augustine told the story of an actor, who in a witty joke said "...that he would say in the theatre ...what all had in their minds, and what all willed ... [and then] with great expectation, all being in suspense and silent, is affirmed to have said: ***Your will to buy cheap, and sell dear***..."

A second Price-Value corollary, derived from the above assertions, contends that:

1c) A sale requires that the buyer's value-estimate (V_b) exceeds that of the seller (V_s) – even though the buyer's initial offer price is below the seller's initial asking price, and the corollary is that the seller' value-estimate (V_s) is less than that of buyer's value-estimate (V_b). Logically: [2] if, per axiom (1a) and corollary (1b), $V_s < P_s < V_b$, then: $V_s < V_b$. Then, in the marketing process, the equilibrium will be met and the price agreed at the Marshallian scissor's intersection[3] where $V_s \leq V_b = P_s$, i.e. where the competing offers in response to the adjusting seller's and buyer's values just meet establishing the market price P_s.

A third Price-Value corollary, derived from the logic in corollary (1c), asserts that:

1d) Value-estimates not only differ from price but, also, differ between buyer and seller.

The above assertions demonstrate that price and value are insufficient to explain a market – an ideal or archetypical value (market-worth) is needed to explain market behaviour – this leads to the second and third theorem in this paper:

2) Market-worth is what sellers and buyers are guesstimating when each sets a value – value-estimates are probability densities with the market nature subsumed in the functional form (normal, log-normal, normal approximation to the binomial, etc.) and the expected risk represented by the standard deviation.

3) Price differs from market-worth – even if drawn from a free-market, arms-length sale, price may be: Greater than, Equal to, or Less than market-worth.

Supporting the third theorem is an axiom drawn from the Price-Value corollary (1c):

3a) **The sales price is derived from within the overlap of a seller's and a buyer's imperfect value-estimates** of market worth (respectively, V_s and V_b) and a transaction is concluded as $V_s \Rightarrow V_b$. (see Figure 1 – the negotiating market behaviour occurs in the area to the left of the equilibrium point where $V_s < V_b$.)

This paper is a deductive theoretical enquiry into how and why market price rises above market-worth (i.e. a price bubble) and the means by which a market eventually corrects from a bubble back to the fundamental of market-worth.

THE WORTHY NOTION OF WORTH

The analysis in this paper uses the concepts of Price and Value and Market-worth (the market-worth is the archetypical value of what is being bought/sold). While the market-worth is usually imperfectly known, rational buyers and sellers seek to estimate *market-worth* in their Value-estimates. The importance of worth can be taken in the observation that St. Augustine neither extolled *Buy cheap and sell dear* as an aphorism of thrift nor saw it as a valid way-of-life but asserted "...such a will is in truth a fault...". If that maxim is expanded to: *buy cheaper than what it is worth and sell dearer than what it is worth*, it clearly celebrates the base, mean traits of sharp-trading, deceit, cupidity, and even fraud.

While a variety of owner-specific circumstances/attributes can alter the fundamentals of worth after an ownership change (scale effect, externalities, aesthetics, opportunity, sub-division, development, etc.), worth tends to be relatively stable, in either absolute or dynamic terms. While, *The Greater Fool Theory* is a non-worth-based-investing logic, it provides scant comfort if there are no fools greater than oneself.[4] Given the assumption of rational-risk-averse investors, worth is a basic fundamental of value estimation.

It is important to note this paper makes the simplifying assumption that the core-worth functions of the buyer and seller are identical—the differences in the buyer and seller Value-estimates are due to risk-induced discount differences arising from information asymmetries – i.e. the seller's knowledge and understanding of the properties attributes are more complete than those of the buyer. Value-estimate differences between buyers are due to differences in perceptions, risk preferences, and other attributes—a similar argument can be made for Value-estimate differences between sellers.

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[2] This contention appears to contradict conventional property valuation theory (Ratcliff, 1949, 1965, and 1979; AIREA .1984; Baum, et. al, 1996) but is, in fact, consistent with it when represented as the seller moving a left-wise in a down-ward direction along the supply curve and a buyer moving left-wise in an up-wards direction along the demand curve, in response to offers and counter-offers towards an equilibrium point at the meeting of the "Marshallian" scissors.[3]

[3] Attributed to economist Marshall, Alfred (1842-1924).

[4] The belief held by one who makes a questionable investment, with the assumption that s/he will be able to sell it later to a bigger fool.

THE MODEL

This paper, being theoretical in nature, affords the comfort of simplifying assumptions and certainty of outcome in its definitions. While price, value, and worth are often seen as synonyms, in this paper, each has a very specific and different meaning:

- P = Price = what is paid for an asset (property, precious metal/gem, fine art, etc.),
- V = Value = an estimate of worth – the price that should be paid for an asset,
- V_s, V_b = the estimates of worth made by, respectively, the seller[5] and the buyer, and
- W = Worth = the *Fisherian* value of an item, as defined by the present value (PV) of all of its net cash flows.[6] While the market participants are assumed to have indefinite estimates of worth, the analysis will, in this theory paper, contrast the price and value-estimates with the definitive worth – e.g. all the cash flows and the discount rates are given.

Modelling Worth

In real estate, a property's future value flows (i.e. cash or equivalent) consist of real or imputed net rents from improvements (gross rent less any operating and/or maintenance expenses), net earnings from rural land (gross farm income less any associated costs of production, harvest, other on-farm items, and transportation out), lifestyle benefits, other psychic-gains, and any capital gain at the time of future sale.

$$W_\Omega = f(Y, g, n, i_r) \quad (1)$$

W = market-worth
Ω = end of period Ω
n = year from start at 0
Y = annual net return at series start
g = real rate of growth in Y
i_r = risk-free discount rate[7]

The functional form of equation (1) is assumed to be:

$$W_\Omega = \sum_{t=\Omega}^{t=\infty} Y(1+g)^{\Omega}[(1+g)/(1+ i_r)]^{(t-\Omega)} \quad (1b)$$

If all expectations of capital gain are solely linked to constant growth in income, the simplifying assumptions in the Gordon Growth Model (Gordon, 1962) are appropriate and it can be used to contract equation (1b) to:

$$W_\Omega = Y(1+g)^{(\Omega+1)}/(i_r - g) \quad (2)$$

Equation (2) describes how market-worth rises over time to reflect the rising net inflows of value to the asset – NB: these rises in market-worth are due to time-value-of-money effects and (similar to the price rise of a zero-coupon bond) should be seen as income, rather than interpreted as a capital gain.[8]

Modelling Value-estimates

If the seller/buyer had perfect information s/he would know the market-worth of what was being sold/bought and both would have the same point Value-estimate – thus, under perfect information:

$$V_s = V_b = W_\Omega \quad (3)$$

However, as noted previously, the value-estimates of the seller (V_s) and buyer (V_b) are attempts to estimate market-worth from imperfect information—these imperfections are represented in the model via normally distributed error terms of, respectively, ϵ_s and ϵ_b with nil means and standard deviations

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[5] It should be noted that V_s (estimate of worth to seller) is distinguished from Ratcliff's (1972) V_s = Subjective Value – which he uses for *both* sellers and buyers as their 'subjectively determined value', to which we ascribe V_s and V_b respectively as their upper and lower limits of value. Ratcliff uses V_b as the buyer's bid price.

[6] Fisher (1906).

[7] Due to the assumption above that "all the cash flows and the discount rates are given", in estimating definitive worth W_Ω.

[8] Many types of assets, including zero-coupon bonds, go through this similar process: Market-values reflect the discounted expected yield-to-maturity; Subsequent market-values are higher because, with the passage of time, a portion of the previously recognized expected value flow is realized; The realization of an expected value flow (e.g. as a periodic payment, lump sum, or price rise) should always be treated as a regular income/(loss). Only an unexpected value flow (from unexpected shifts in: interest rates, investor confidence, asset risk, etc.) should be treated as a capital gain/(loss).

of, respectively, σ_s and σ_b. In recognition of their imperfect knowledge, the seller and buyer, adjust for the associated risk by increasing their respective discount rates to, respectively, i_s and i_b.

Given that a seller tends to have more experience with the asset being sold/bought than a buyer, logically:

$$\sigma_s < \sigma_b \quad (4)$$

$$i_s < i_b \quad (5)$$

Sellers and buyers are limited in the information that they can access when they seek to operationalise their value-estimates of worth:

$$V_{s\Omega} = f(Y, i_s, P_\Omega, P_{\Omega-1}, P_{\Omega-2}, P_{\Omega-3}, \ldots P_{\Omega-n}, W_\Omega, Q, K_s) \quad (6)$$

$$V_{b\Omega} = f(Y, i_b, P_\Omega, P_{\Omega-1}, P_{\Omega-2}, P_{\Omega-3}, \ldots P_{\Omega-n}, W_\Omega, Q, K_s) \quad (7)$$

P_Ω = price at the end of period Ω;
NB: in equilibrium: $P_\Omega - P_{\Omega-1} = 0$
Q = Quantity
K_s = stock of what is being sold

Sellers and buyers are assumed to include in their valuation the annual earnings (or the equivalent) and any expected future capital gains—as the later are difficult to estimate, a backward-looking rational expectations approach was assumed (Dornbusch and Fischer, 1993) using a one year myopic estimate and a rolling three-year average (with declining weightings over the three years). Unlike annual earnings, capital gains cannot rationally be expected to continue forever—thus, the model uses a multiplier to capture expected duration (e.g. three to 20 years) and uses sales quantity divided by total available stock to adjust for perceived risk (e.g. risk is higher when sales are lower). Differing perceptions and personal attributes among the sellers and the buyers are recognized by having the value-estimates centred on a mean value with a normally distributed error term:

$$V_{s\Omega} = Y(1+g)^{(\Omega+1)}/(i_s - g) + \pi \pm \epsilon_s \quad (6a)$$

π = Present value of the capital gain expectations[9]

$$V_{b\Omega} = Y(1+g)^{(\Omega+1)}/(i_b - g) + \pi \pm \epsilon_b \quad (7a)$$

In equilibrium, the capital gain expectations are, by definition, nil (i.e. $P_\Omega = P_{\Omega-1} = P_{\Omega-2} = P_{\Omega-3} = \cdots = P_{\Omega-n}$) and equations (6a) and (7a) reduce to:

$$V_{s\Omega} = Y(1+g)^{(\Omega+1)}/(i_s - g) \pm \epsilon_s \quad (6b)$$

$$V_{b\Omega} = Y(1+g)^{(\Omega+1)}/(i_b - g) \pm \epsilon_b \quad (7b)$$

In a dynamic adjustment situation, with myopic (one-year) backward-looking rational expectations, the capital gains expectations are defined by:

$$\pi = \gamma P_\Omega(1 - P_{\Omega-1}/P_\Omega)(\alpha W_\Omega/P_\Omega)^\varphi(1 + i_r)^{(1/\psi)} \quad (8)$$

α = scaling parameter
γ = duration parameter ($3.00 \le \gamma \le 20.00$)
φ = slope parameter
ψ = discount period

The term W_Ω/P_Ω, in equation (8) exponentially *damps* capital-gains expectations, as the fraction of *the stock being sold* falls—it is irrelevant in equilibrium, because (by definition) the capital-gains expectations are nil in equilibrium.

In a dynamic adjustment situation with backward-looking rational expectations based on three years, equation (8) becomes:

$$\pi = \gamma P_\Omega(1 - .5P_{\Omega-1}/P_\Omega - .3P_{\Omega-2}/P_{\Omega-1} - .2P_{\Omega-3}/P_{\Omega-2})(\alpha W_\Omega/P_\Omega)^\varphi(1 + i_r)^{(1/\psi)} \quad (8a)$$

The sum of the weightings in equations (8a) equal one—the weightings[10] reflect the idea that recent years have a greater effect on capital gains expectations than earlier years.

Modelling Market Behaviour

The value-estimate variability represented in the error terms in equations (6a) and (7a) drives the

[9] This part of the model is consistent with property valuation practice, but is flawed—it double counts the capital returns from rises in annual income (i.e. once in the Gordon growth part of the equation and again as a price rise that increases expectations of future capital gains). As long as π is nil, double counting is theoretical rather than a real issue.

[10] These need to be established by empirical analysis but are logically in descending order with greatest weight on the most recent period and declining over the number of backwards looking periods.

market. Specifically, sales require an overlap in the above distributions and the market-clearing price occurs at the price that sets equation (6a) equal to equation (7a).

In equation (6a), as $Є_s$ approaches negative infinity, the fraction of sellers active in the market (Q_s/K_s) approaches nil; when $Є_s$ equals nil, half the potential sellers are active in the market; as $Є_s$ approaches infinity, all sellers are active in the market. A similar but opposite effect occurs in equation (7a) for buyers as $Є_b$ rises from negative infinity to nil to infinity (see Figure 1).

The Cumulative-Normal-Density functions for $Є_s$ and $Є_b$ are prohibitively complex, but approximated in shape and character by a variant of the *Sigmoid* form of the Logistic Curve (Wikipedia, 2007b).

$$M_s = 1/(1 + e^{-hkЄ_s}) \quad (9)$$

$$Є_s = -ln(1/M_s - 1)/(hk) \quad (9a)$$

e = exponent
ln = natural log of (...)
Ms = fraction of total sellers active
h = a scaling parameter
k = a slope parameter (related to σ_s)

$$M_b = 1/(1 + e^{hjЄ_b}) \quad (10)$$

$$Є_b = ln(1/M_b - 1)/(hj) \quad (10a)$$

Mb = fraction of total buyers active
j = a slope parameter (related to σ_b)

Once π (expectation of capital gain) is incorporated into the valuation equations, the Gordon growth effects should be removed to avoid double counting the income related capital gains.[11] Thus, assuming sellers and buyers are aware of the double counting risk, removal of the Gordon growth effects from equations (6a) and (7a) changes them to:

$$V_{sΩ} = P = Y(1+g)^{Ω}/i_s + π - ln(1/M_s - 1)/(hk) \quad (11)$$

$$V_{bΩ} = P = Y(1+g)^{Ω}/i_b + π + ln(1/M_b - 1)/(hj) \quad (12)$$

Equations (11) and (12) can be changed and reorganized to:

$$M_s = 1/[1 + e^{hk([Y(1+g)^{Ω}]/i_S + π - P)}] \quad (13)$$

$$M_b = 1/[1 + e^{hj(P - π - [Y(1+g)^{Ω}]/ib)}] \quad (14)$$

Because there is no reason why the total potential supply of what is being sold has to, or should, equal the total potential demand for it, equations (13) and (14) need to be modified before they can be used as, respectively, the supply curve and the demand curve:

$$Q_s = K_sM_s = K_s/[1 + e^{hk([Y(1+g)^{Ω}]/i_S + π - P)}] \quad (15)$$

$$Q_d = D_bM_b = D_b/[1 + e^{hj(P - π - [Y(1+g)^{Ω}]/ib)}] \quad (16)$$

Q = Quantity
K_s = capital stock of what is being sold
D_b = potential total demand

$$Q_{mkt} = Q_s = Q_d \quad (17)$$

A range of prices are substituted into Equations (15) and (16), to create, respectively the supply and demand curves.

Where the conditions in equation (17) are met the market equilibrium price is achieved.

NUMERIC EXAMPLES

Under Equilibrium

In the real world, the forgoing flows and the appropriate discount rates are subject to massive uncertainty that is beyond the scope of this enquiry. This study simplifies the analysis by assuming that the:

a) Inflation/deflation is nil,

[11] The empirical issue of whether or not sellers, buyers, appraisers, and researchers are actually avoiding double counting i*ncome-related expectations of capital gain* is an important issue that future research should seek to resolve—i.e. are equations 11 and 12 more appropriate than equations (6a) and (7a).

b) All sellers/(buyers) are identical, except for the differences subsumed in ϵ_s/(ϵ_b).
c) All items being sold/(bought) are identical
d) The parameters are: (see Table 1 - next page)

Table 1: Parameter description and assumed values, under equilibrium

Description	Symbol	Value
1) Real rate of growth in Y	g	1.0 %
2) Risk-Free Discount Rate	i_r	5.0 %
3) Sellers' Discount Rate	i_s	6.0 %
4) Buyers Discount Rate	i_b	7.0 %
5) Scaling Parameter	h	.00001
6) Scaling Parameter	α	0.60
7) Duration Parameter	γ	10.0
8) Slope Parameter (related to σ_s)	k	3.0
9) Slope Parameter (related to σ_b)	j	4.0
10) Slope Parameter	φ	2.3
11) Discount Period	ψ	10
12) Wasting or Decay Rate of the Asset Stock	ω	3.0 %
Assumed Starting Values for Key Variables	**Symbol**	**Value**
a) Capital Gain Expectations	π	0.0
b) Annual Price Changes	$P_\Omega = P_{\Omega-1} = P_{\Omega-2} = P_{\Omega-3} = ... = P_{\Omega-n}$	
c) Annual Net Return at t = 0	Y	$35,000
d) Potential Total Sales	Ks	10,000,000
e) Potential Total Purchases	D_b	11,000,000

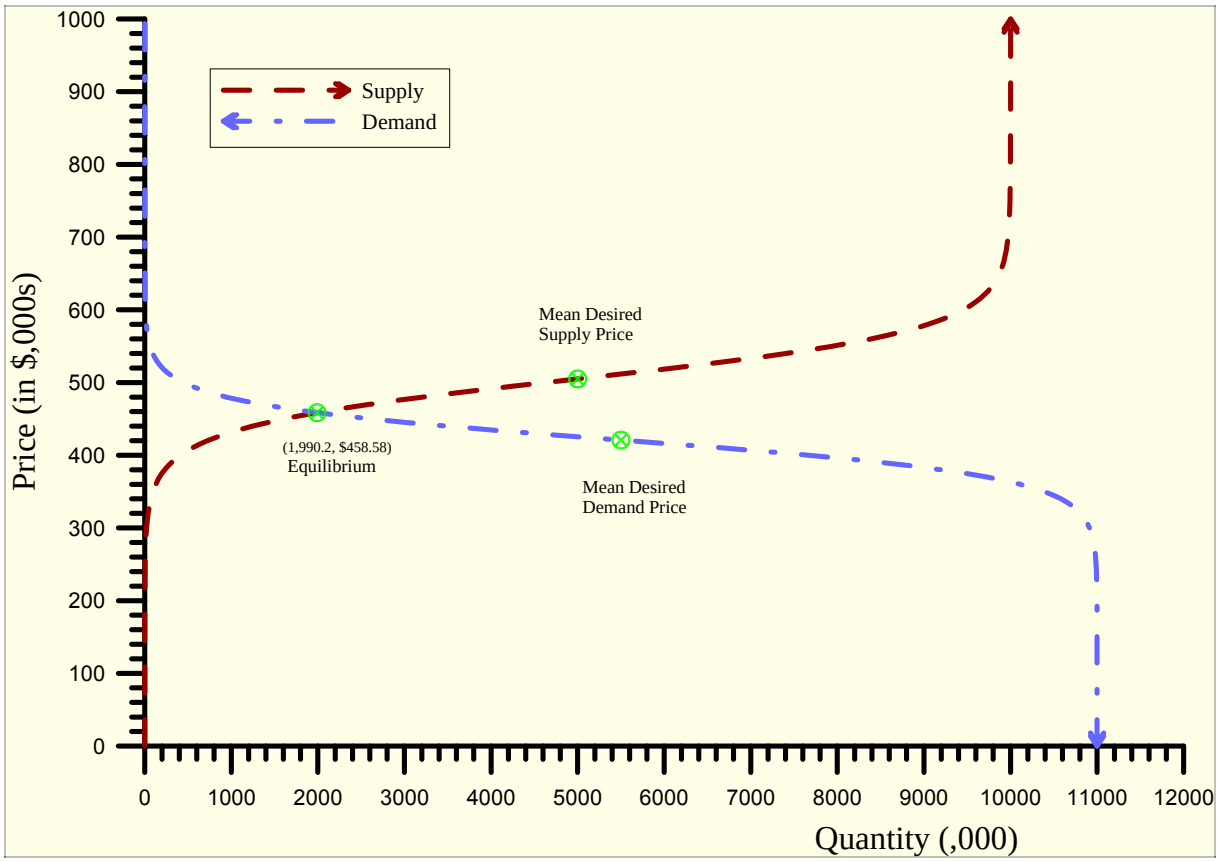

Figure1: Market supply and demand

In Figure 1, the Equilibrium Price is between the Mean-Desired-Supply Price and the Mean-Desired-Demand Price and the Equilibrium Quantity is to the left of the quantities associated with those points. This reflects three attributes found in most markets:

1) Most sellers are reasonably "bearish" about the market,
2) Most buyer are reasonably "bullish" about the market, and
3) The total potential supply and demand are reasonably balanced.

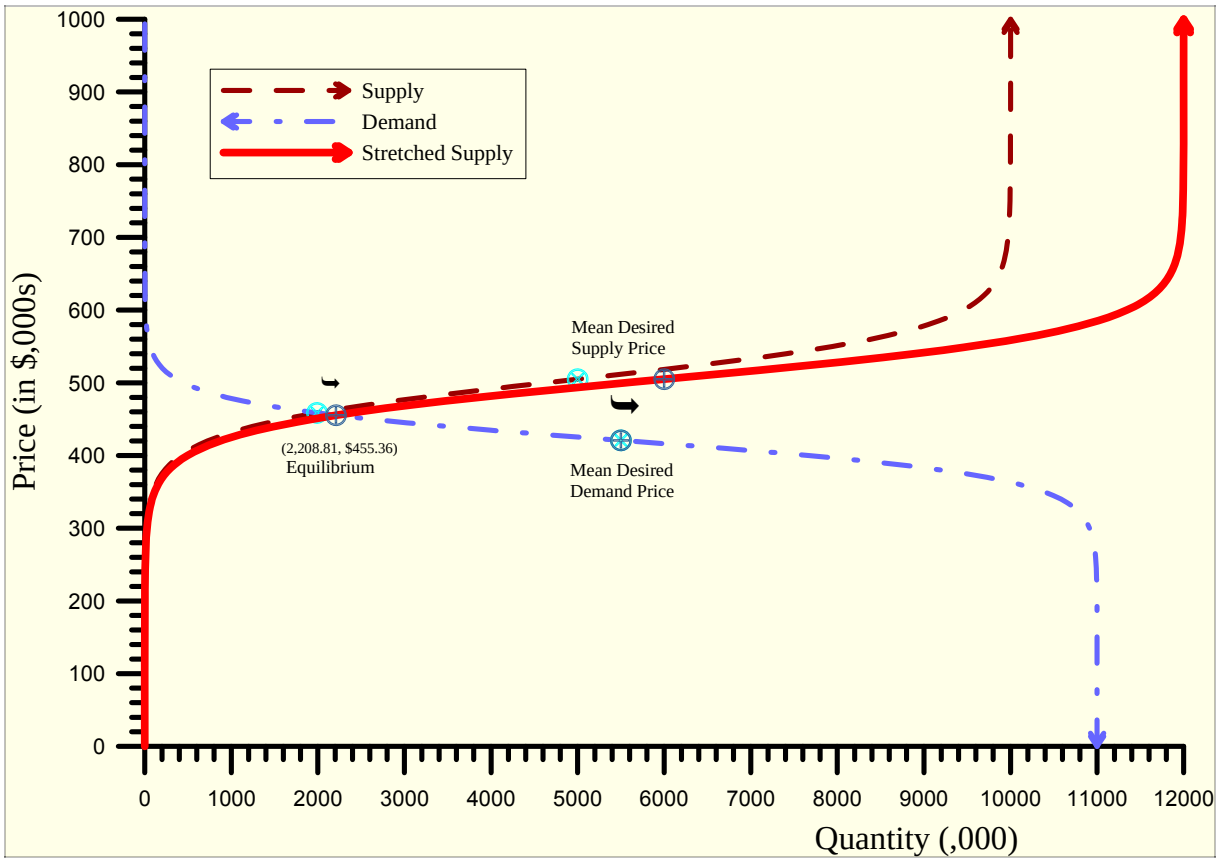

Figure 2: market supply & demand, after a 20% increase in supply

Increasing the total potential supply shifts the anchor point (quantity when P = $∞) in Figure 1 to the right which rotates the supply curve down around the origin and shifts the Equilibrium along the Demand Curve. Given the relatively elastic nature of demand over the vast majority of the feasible demand, large increases/(decreases) in total potential supply will, *ceteris paribus*, generate relatively small decreases/(increases) in Equilibrium Price. Similarly, large increases/(decreases) in total potential supply will, *ceteris paribus*, generate relatively small increases/(decreases) in Equilibrium Price. Given the assumptions in this model, if government policies are implemented that generate a 20 percent increase in the housing stock, a long-run equilibrium will form at a price that is 0.70 percent below the earlier equilibrium price and at a quantity that is 11.0 percent above the earlier equilibrium quantity (Figure 2).

If policies are implemented that generate a 20 percent decrease in the housing stock (e.g. mould causes a large number of houses to be abandoned as unfit for human habitation), a long-run equilibrium will form at a price .85 percent above the equilibrium price in Figure 1 and at a quantity that is 12.2 percent lower (Figure 3).

Figures 2 and 3 indicate that government fiscal policy directed at affecting the total supply (K_s) has little effect on the long-run equilibrium housing price—fiscal policy rotates the supply curve up or down around the origin and, thus, has little effect on the Y-axis prices. Similarly, a large rise or fall in the total demand (D_b) due to net immigration or migration will have little effect on the long-run equilibrium housing price — unless immigrants change the core-worth perspective in the buyer Value-estimate function (i.e. that would shift the demand curve, rather than rotate it around the Y-axis infinity point).

Monetary policy effects are examined by shifting the discount rates (i_s and i_b) up by 20 percent—continuing with the assumption that buyers and sellers are identical except for the differences subsumed in i_s, i_b, $Є_s$, and $Є_b$, it is assumed that a monetary policy shift in r affects i_s and i_b by an equal percentage shift. Monetary policy is much blunter in its targeting than fiscal policy—it affects both supply and demand.

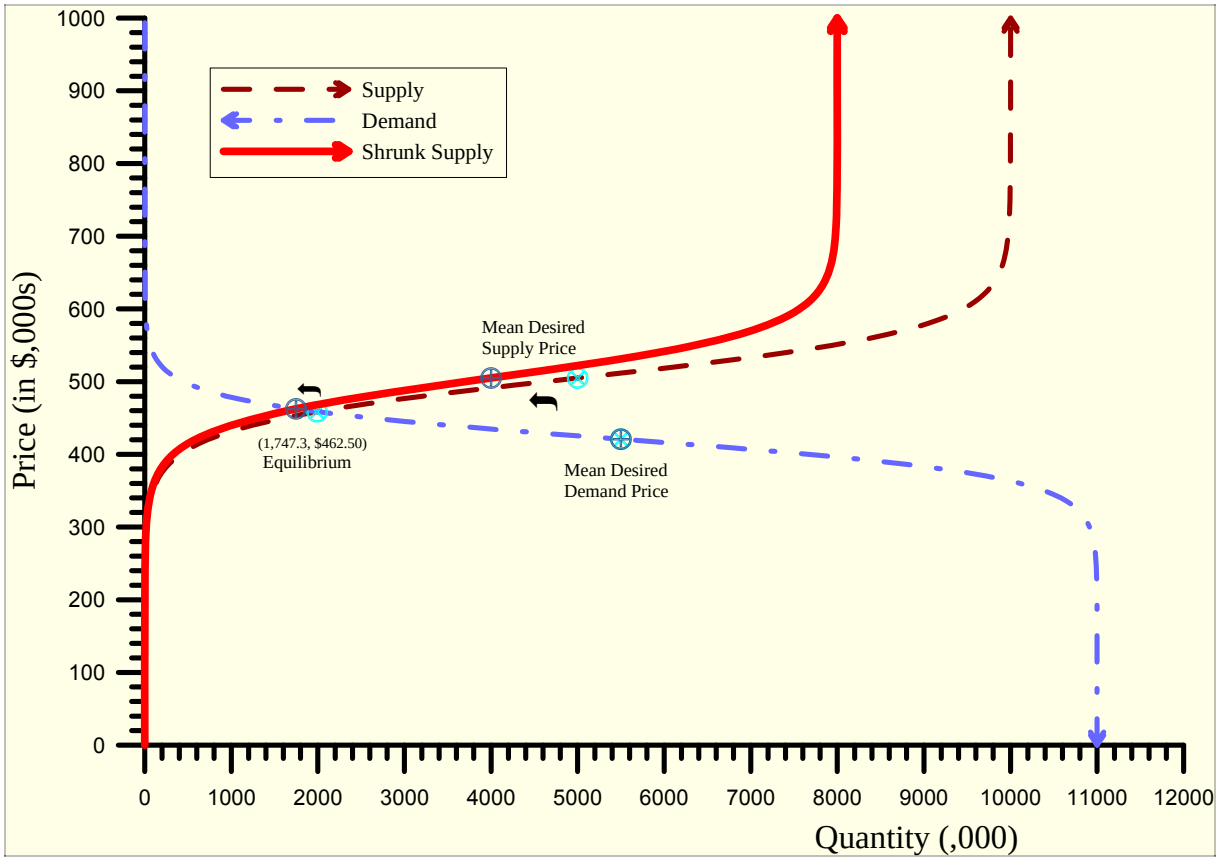

Figure 3: Market supply & demand after a 20% decrease in supply

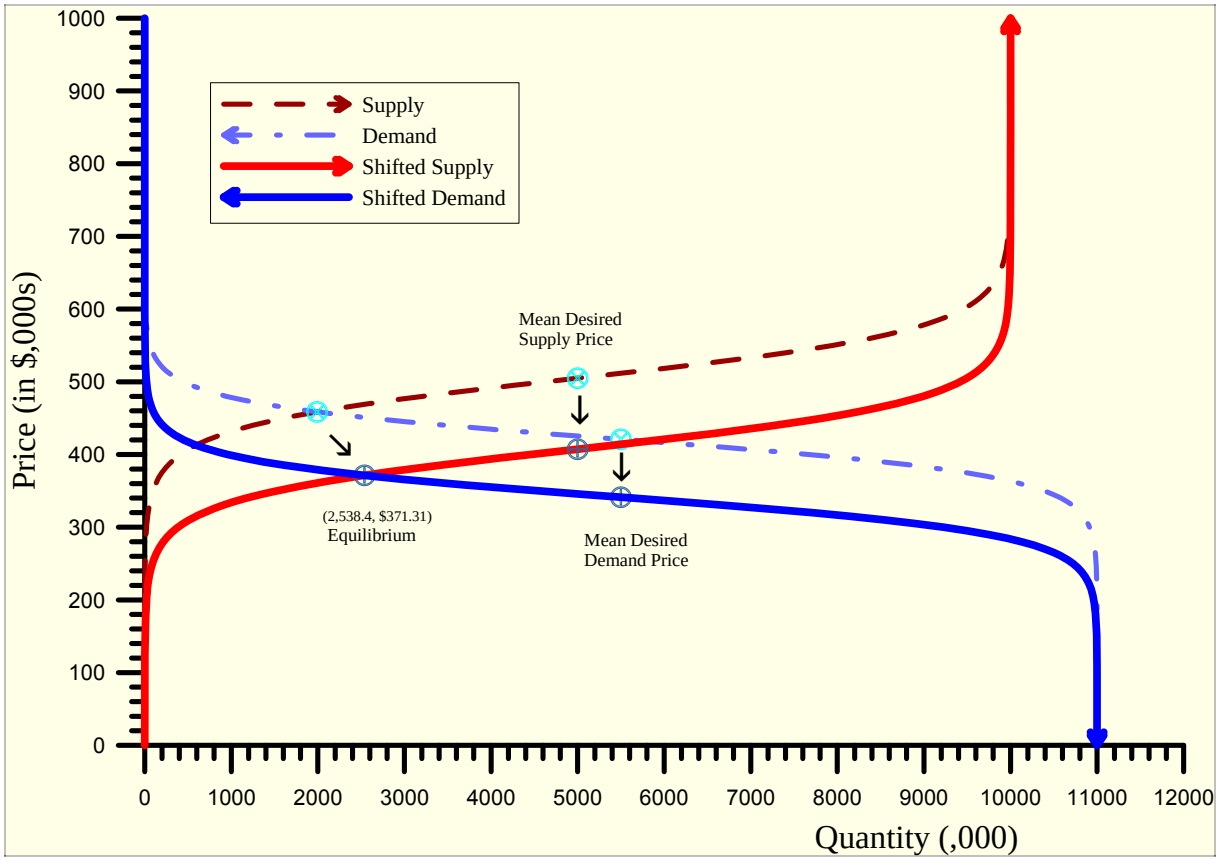

Figure 4: Market supply & demand after a 20% rise in interest rates

Figure 4 shows that, given the model assumptions, a 20 percent rise in the seller and buyer discount rates will shift prices down by 19.0 percent (from $458,580 to $371,310) and will increase output by -27.5 percent (2,013,300 units to 1,495,600 units).

Figure 5 shows that, given the model assumptions, a 20 percent fall in the seller and buyer discount rates will shift prices up by 24.9 percent (from $458,400 to $599,990) and will decrease output by 37.3 percent (2,013,300 units to 1,262,400 units).

Based on Figures 4 and 5, monetary policy should be a powerful and effective means to manage the long-run-equilibrium housing price.

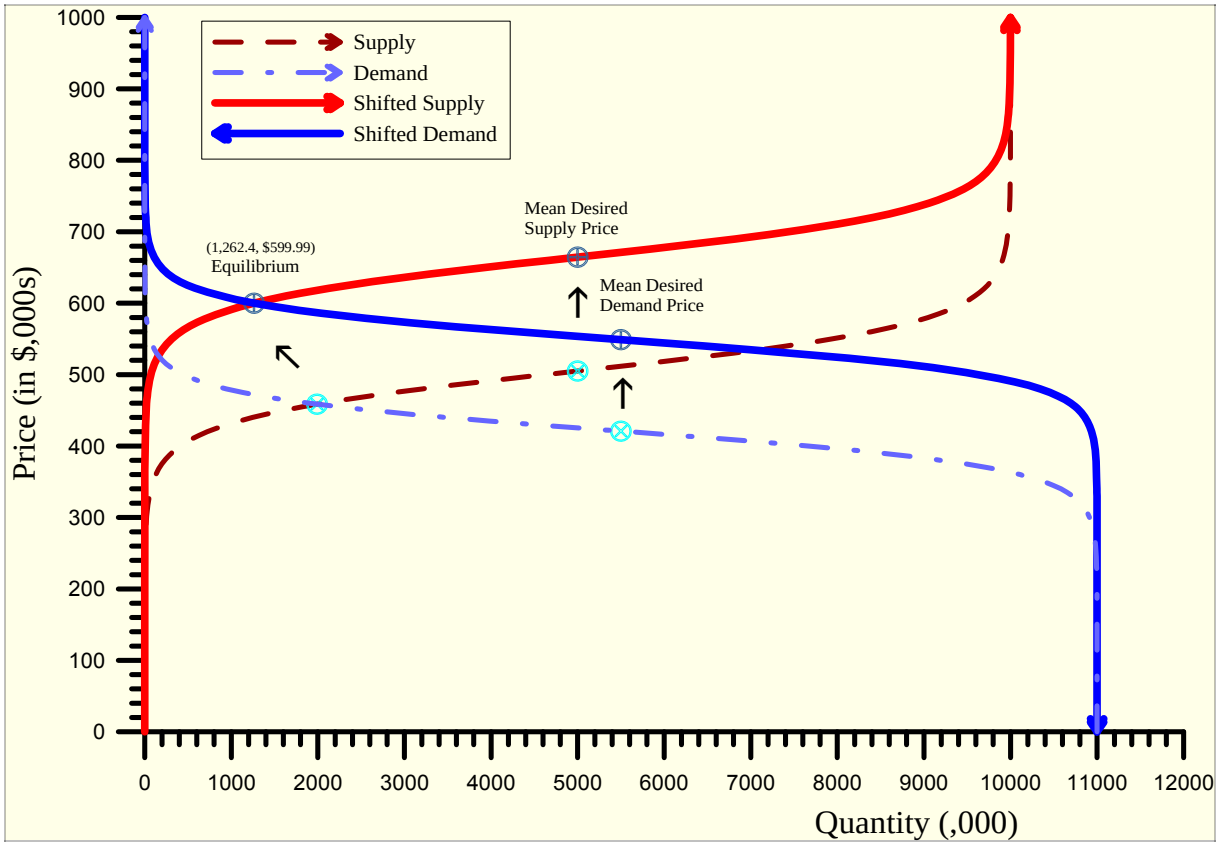

Figure 5: Market supply & demand after a 20% fall in interest rates

Dynamic Market Solution – Myopic Backward-looking Rational Expectations

While long-run equilibrium outcomes in Figures 2 through 5 are interesting, as Keynes (1923, ch.3, p.65) observed "...*in the long run we are all dead*…" Thus, to be relevant to living voters, policy affects should also consider short- and intermediate-run dynamics. This model becomes dynamic if the equilibrium assumption of "π = nil" is relaxed.

A dynamic view is approximated in this study by using equations (15) and (16) to formulate partial market equilibriums monthly with the P of the current month becoming the P_{Ω} of the next month and the $P_{\Omega-1}$ of the 12th month following the next month—this gives a 12-month rolling average price rise.

$$Q_s = K_s M_s = K_s/[1 + e^{hk([Y(1+g)^{\Omega}]/i_s + \pi - P)}] \quad (15)$$

$$Q_d = D_b M_b = D_b/[1 + e^{hj(P - \pi - [Y(1+g)^{\Omega}]/i_b)}] \quad (16)$$

Under myopic (one year) backward-looking rational expectations, the expectations for future capital-gains are defined by (see equations (6c) and (7c), above):

$$\pi = \gamma P_{\Omega}(1 - P_{\Omega-1}/P_{\Omega})(\alpha W_{\Omega}/P_{\Omega})^{\varphi}(1 + i_r)^{(1/\psi)} \quad (8)$$

The model is a cranky iterative process of searching for a market clearing price:

$$Q_{mkt} = Q_s = Q_d \quad (17)$$

Equation (8) embodies a simple but powerful actuarial/accounting/finance tenet that: *In the absence of evidence to the contrary, the best predictor of the future is the past* (Committee on International

Accounting, 1975; Niehaus and Terry, 1993; AASB, 2004, pp. 13-18). The following *turn-around* equations were added to make the model implosive (i.e. by limiting the minimum and maximum prices):

$$@if(P < 0.40W_{\Omega}, \pi:=-0.10\pi) \quad (20)$$

$$@if(P > 0.90W_{\Omega}, \pi:= 0.10\pi) \quad (21)$$

Equations (20) and (21) give the model prices a chaotic sine-curve shape (see Figure 6). In Figure 6, the lower risk-free discount rate causes the perfect-market-worth function to be well above the long-run market-worth function.[12]

The back-ward-looking model is transformed from a one-year to a three-year cycle by using equation (8a) in place of (8):

$$\pi = \gamma P_{\Omega}(1 - .5P_{\Omega-1}/P_{\Omega} - .3P_{\Omega-2}/P_{\Omega-1} - .2P_{\Omega-3}/P_{\Omega-2})(\alpha W_{\Omega}/P_{\Omega})^{\varphi}(1+ i_r)^{(1/\psi)} \quad (8a)$$

It is clear in Figures 6 and 8 that the present value of the expected capital gains are a destabilising influence—i.e. drives the market-price cycle from the long-run market-worth trend. The lagged-effect of past price gains on current gains is apparent in Figures 7 and 9.

In Figure 9, the price cycle appears to display a rapid large up-tick followed a series of echoes that diminished and eventually become negative until there is another rapid large up-tick in price, and so forth.

In the controlled set of circumstances that resulted in Figures 6 through 9, the Gordon Growth Model predictions are very much more stable and realistic than the chaotic and the alternately overly optimistic to pessimistic results of using past price gains in a backward-looking prediction model. Past-price-based valuation models appear to be more of a game of chance than a sound investment input.

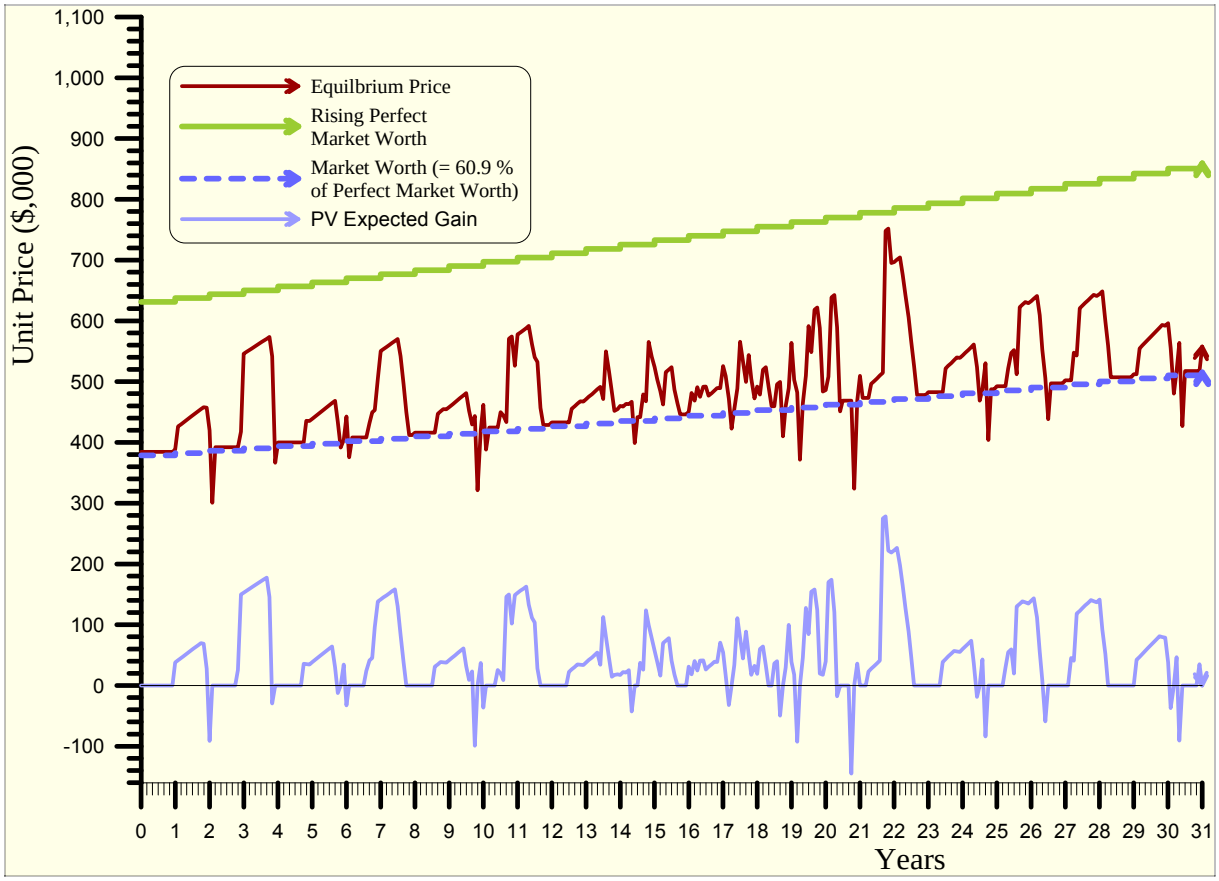

Figure 6: Worth and equilibrium price over time with a one-year backward-looking expectation

[12] See Gleick, 1988, for a detailed discussion of how chaotic outcomes can arise from combining simple non-linear equations into dynamic models. In particular p. 92-94, the *Noah Effect* – explains *discontinuity*: when a quantity changes, it can change almost arbitrarily fast. Also, the *Joseph Effect* explains *persistence* (of famines or, in our case, "property booms, and busts") that can vanish as quickly as they come.

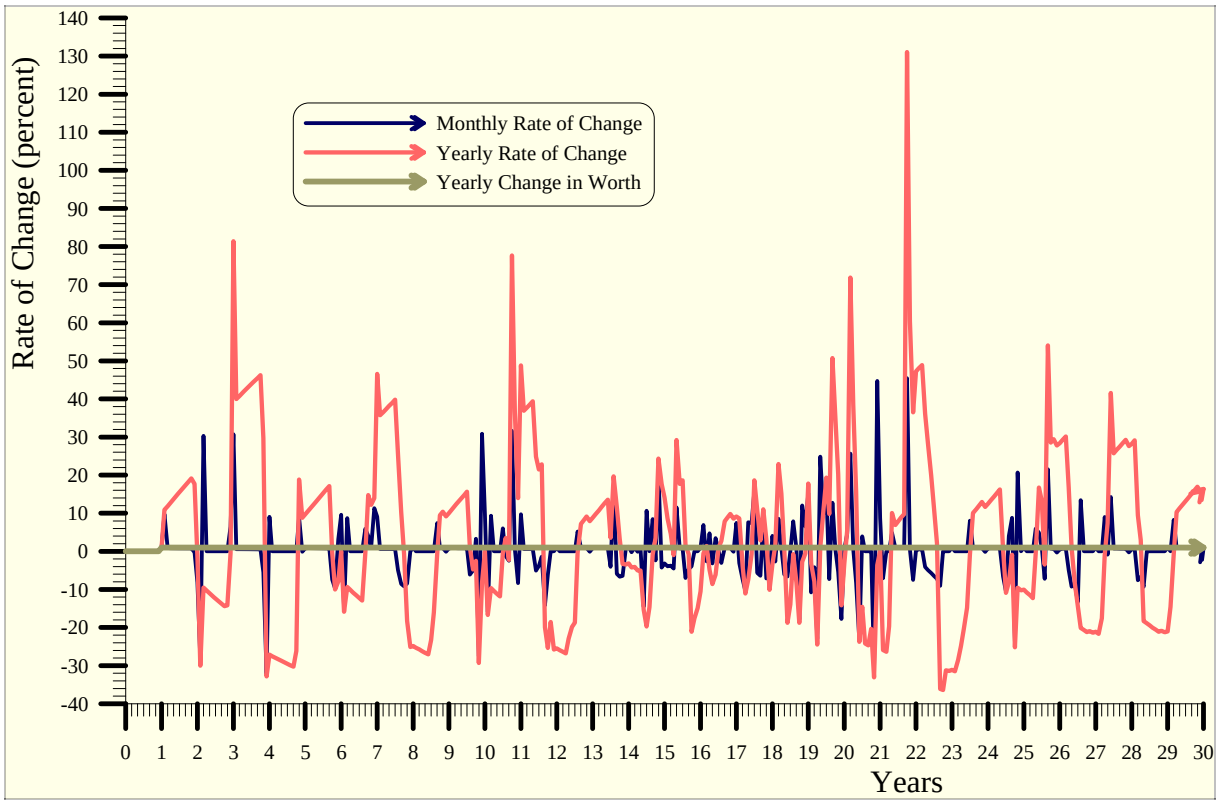

Figure 7: Price and worth change over time with a one-year backward-looking expectation

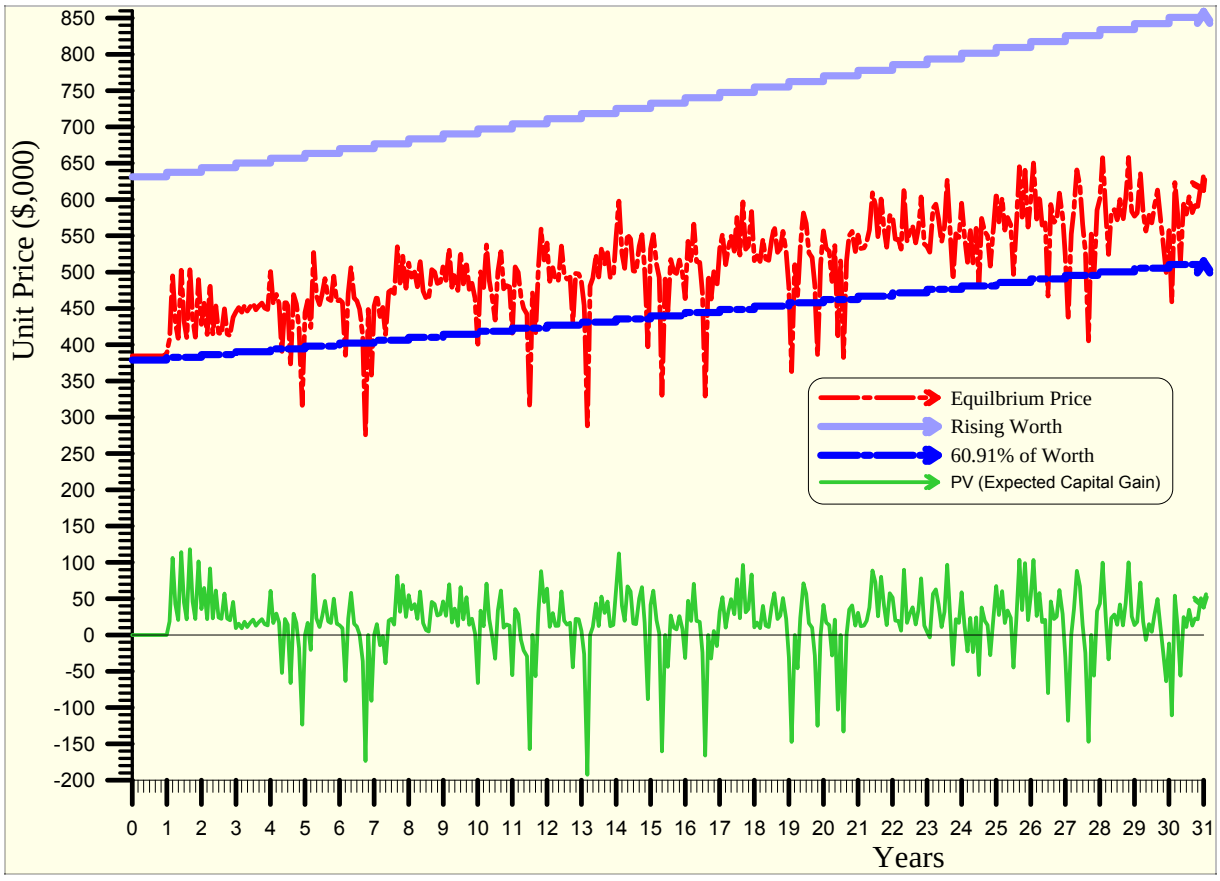

Figure 8: Price and worth change over time with a one-year backward-looking expectation

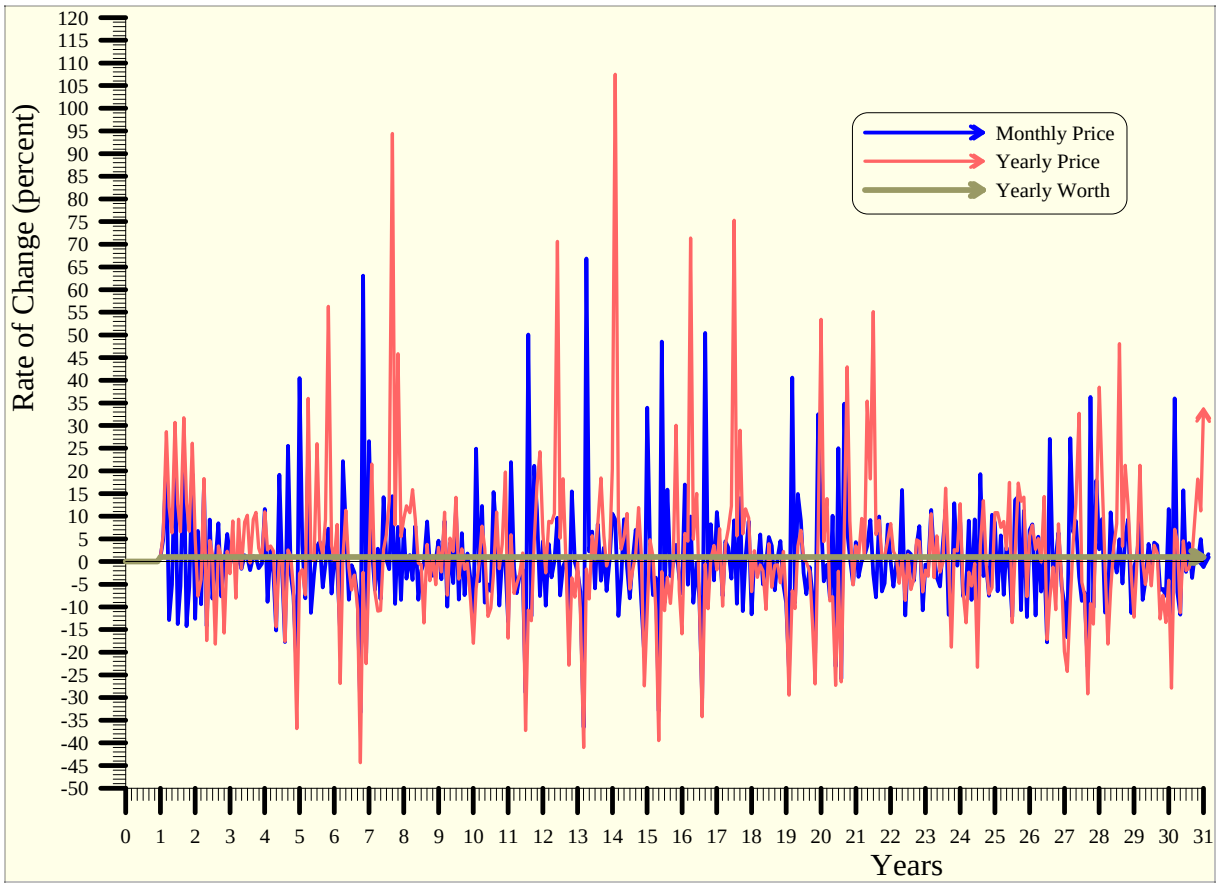

Figure 9: Price and worth change over time with a thee-year backward-looking expectation

CONCLUSIONS

Buy cheap and sell dear is ancient investment advice that traces back to Aristotle (350 BCE)—who attributed to *Thales the Milesian* (6th Century BCE). The cynicism of this advice is evident in its difficulties to implement and in its failure to consider motives other than profit. The key findings in this study are:

A) General Logic:

- Price differs from Value—Price may be equal, greater than, or lesser than Value,
- A sale requires that the buyer's value estimate exceed that of the seller (i.e. buyers tend to be more optimistic than sellers—buyers are bulls and sellers are bears),
- Price differs from Worth—Price may be "Greater than, Equal to, or Less than Worth",
- The sales price is derived from within the overlap of a seller's and a buyer's imperfect value-estimates of market worth ($P = V_s = V_b$), and
- *Buy cheap and sell dear* ultimately simplifies to the non-worth-based-investing logic of: *The Greater Fool Theory*.

B) Equilibrium Market Analysis:

- Prices are set on the margin (i.e. by active sellers and buyers) and are little affected by total supply,
- Even large shifts in supply have relatively little effect on the equilibrium (market) price and quantity sold—governments should not expect to be able to lower housing prices by adding reasonable numbers of units to the housing stock,
- Changes in the interest rates should have significant effects on the equilibrium (market) price and quantity sold—thus, governments should be able to manage housing prices by altering the interest rate. NB: An interest rate housing policy tool will be blunted if other policies are buffering the effective interest rates experienced by the buyers and sellers (e.g. if negative gearing is available to housing owners or if depreciation is deductible for rental units but is not recaptured

as regular income when the house is sold).

C) Dynamic Market Analysis:

- Incorporating backward-looking gains information into a property pricing model tends to destabilize the market—the resulting *Chaotic* pricing turns a property market into more of a gambling arena than an investing activity.
- A three-year backward-looking price-expectations model tends to be more Chaotic than a one-year model and both are more prone to over- and under-estimating prices than a rational expectations model.

Lags in the backward-looking price model mean that, by the time the model indicates it is time to buy, the turn-around has already occurred and the best time to buy may have passed.

Future Research:

The basic models predictions outlined in this study need to be tested against empirical data in a variety of markets—where possible the parameter values in the model should be estimated from empirical data. Also, the models' sensitivity should be examined by testing the effects of varying key parameters on the model outcomes. The empirical issue of whether or not sellers, buyers, appraisers, and researchers avoiding the double counting of i*ncome-related capital-gain expectations* is an important issue that should be resolved in future research.

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