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Bayes Stein Estimators & International Real Estate Allocation

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Abstract

This paper re-examines the issue of international diversification in real estate securities and attempts to address the problem of estimation error in the inputted parameters through the use of two alternative techniques. The Bayes-Stein approach advocated by studies such as Jorion (1985) is used in additional to a simple minimum variance strategy. The results see an increased stability in calculated portfolio allocations in comparison to the classical mean-variance tangency approach, and see significant improvements in ex-post performance. In addition, the minimum variance portfolio significantly outperformed a naive equally weighted strategy ex-post.

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Introduction

A growing literature has emerged examining the potential diversification opportunities that can arise from diversifying internationally in real estate securities. Papers such as Eichholtz (1996), Liu & Mei (1998) and Stevenson (2000a) have examined this issue, finding generally supporting evidence as the attractiveness of foreign investment. Eichholtz (1996) compared the relative benefits of diversifying internationally into both real estate securities and equities, finding that real estate stocks provided greater diversification opportunities. Liu & Mei (1998) also found that property stocks provided some degree of incremental diversification benefits on an international scale. While the authors reported that currency fluctuations accounted for a larger proportion of return variability in comparison to common stocks, even if the currency risk is hedged real estate firms do provide incremental diversification benefits.

Stevenson (2000a), while finding contrary evidence as the relative attractiveness of real estate versus equities, did find that investing internationally in real estate firms provided statistically significant improvements in performance when compared to an all domestic portfolio. The results were also consistent across the ten countries examined when local returns were used, under an assumption of perfect hedging ability. However, when the assumption was made that the portfolio manager did not partake in a hedged strategy, significant results were only obtained for three of the ten markets analysed[1]. The only other proviso with regard to this study was that the gains became insignificant if the international allocation in the portfolio was constrained[2].

Despite the generally supportive nature of the empirical studies to have examined international diversification in real estate securities, all of the existing studies have largely relied on standard mean-variance asset allocation procedures, with little regard to the potential problems in using such a technique[3]. This paper attempts to address two of the key issues concerned with mean-variance optimisation, namely the sensitivity of the estimated allocations to the inputted parameters and the ex-post performance of the optimal portfolios. The issues are highly related and are jointly concerned with the problem of estimation error. Unconstrained standard mean-variance analysis tend to produce relatively 'undiversified' estimated allocations. As Michaud (1989) states, optimisation models are in effect 'error maximisers', producing higher estimated allocations to those securities or assets with relatively high mean returns and low risk measures. Likewise, assets with relatively low returns and high risk measures will have low estimated allocations. The result is that standard procedures often result in corner solutions, and in part due to the undiversified nature of them, generally perform poorly on an ex-post basis[4]. In addition to the problem of undiversified optimal portfolios, standard optimisation models do not take account of the fact that the inputted parameters are themselves subject to estimation error, and that estimated allocations are extremely sensitive to variations in the parameters. Studies such as Kalberg & Ziemba (1984) and Chopra & Ziemba (1993) have found that the estimated allocation are particularly sensitive to variations in the means. In addition, papers such as Jorion (1985) have found that despite seemingly large differences in mean returns, it is not possible to reject the null hypothesis that the returns are equal to zero.

A simple and, for a portfolio manager, practical method of reducing estimation error is to constrain the allocations, thereby forcing greater spread across the assets examined. Papers such as Frost & Savarino (1988) and Chopra (1993) have both used this technique to obtain a greater degree of diversification, while Stevenson (1998, 2000b) analysed the role of the direct real estate market in multi-asset portfolios in a constrained environment. One of the major problems with the use of constraints is that the choice of constraints is at best arbitrary, leading to the results being hard to generalise. This paper therefore examines a further alternative method of reducing estimation error, namely the use of the Bayes-Stein shrinkage approach.

This study analyses indirect real estate security data from eleven countries over the period 1976-1998. The empirical analysis takes three primary perspectives. Initially, the impact of variations in the inputted parameters is assessed, with the analysis then turning to examining the use of Bayes-Stein estimators. Initially optimal portfolios are constructed using two alternative methods and the ex-post performance of the portfolios is then assessed. The remainder of the paper is laid out as follows. Initially, a brief discussion of the Bayes-Stein approach is discussed, while the following section provides details of the data used and methodological framework adopted. The final two sections report the findings of the empirical analysis and provide concluding comments respectively.

Bayes-Stein Estimators

The use of Bayes-Stein estimators is designed to reduce the degree of estimation error and furthermore, decrease the tendency for asset allocation studies to arrive at corner solutions. A further advantage to the use of such estimators is that empirical evidence, such as Jorion (1985) and Chopra, Hensel & Turner (1993) have provided evidence that the ex-post performance of optimal portfolios improves substantially. Jorion (1985) examined seven world equity markets, finding that the Bayes-Stein estimated portfolios significantly outperform the standard MVA tangency portfolio. Chopra, Hensel & Turner (1993) find similar results using a 60 month rolling period strategy and a sample consisting of six equity markets, five bond markets and five cash markets. Additionally, due to the increased stability in allocations obtained, the improvement over the classical mean-variance approach is further enhanced when transaction costs are incorporated into the analysis[5].

The premise behind the Bayes-Stein approach is that due to the sensitivity if the estimated allocations to variations in the parameters, and to relatively extreme inputs, the means of the assets are ‘shrunk’ towards a global mean. This effectively reduces the difference between extreme observations, thus aiding in the attempt to reduce estimation error. The general form for the estimators can be defined as follows:

$$E\left(r_i\right) = w\bar{r}_g + (1-w)\bar{r}_i \tag{1}$$

Where $E(r_i)$ is the adjusted mean, $\bar{r}_i$ is the original asset mean, $\bar{r}_g$ the global mean and w the shrinkage factor. Jorion (1985, 1986) shows that the shrinkage factor can be estimated from a suitable prior as follows:

$$\hat{w} = \frac{\hat{\lambda}}{\left(T + \hat{\lambda}\right)} \tag{2}$$

$$\hat{\lambda} = \frac{(N+2)(T-1)}{\left(\mathbf{r} - r_0\mathbf{1}\right)' S^{-1}\left(\bar{\mathbf{r}} - r_g\mathbf{1}\right)(T-N-2)} \tag{3}$$

Where T is the sample size and S is the sample covariance matrix. Chopra, Hensel & Turner (1993) use a slightly different approach in their analysis. The calculate the optimal portfolios under three alternative scenarios designed to reduce estimation error. Firstly, the sample means of the assets used are assumed to be all equal to the global means for stocks, bonds and cash. The second scenario then also adds in the constraint that the correlation's are equal within each grouping, while the third adds the further constraint that the within group variances are equal. This final scenario effectively reduces the analysis to a three-asset case of stocks, bonds and cash.

The first scenario which assumes equal means is equivalent to analysing the minimum variance portfolio, rather than the tangency portfolio that is more commonly examined, as the estimated allocation are based purely on the variance and covariance terms. This is a scenario used by papers such as Jobson, Korkie & Ratti (1979) and Jobson & Korkie (1981). Jorion (1985) argues that unless all of the assets examined are within the same risk class, such a strategy is hard to reconciles with the idea that a risk-return trade-off exists. While such a strategy is an extreme case of shrinkage, it is examined in this current study for a number of reasons. Firstly, Jorion's (1985) argument on this point is limited in its relevance as all of the assets used are indices of real estate securities. Secondly, the use of the minimum variance portfolio eliminates the largest potential cause of estimation error, namely the mean from the analysis, as the portfolios are determined purely by the variances and covariances. Thirdly, empirical evidence, such as Chopra, Hensel & Turner (1993) and Stevenson (1999) provide strong evidence as to the attractiveness of the strategy.

Stevenson (1999) analysed a total of 38 international equity markets including 15 emerging markets. Due to the non-normality present in emerging market returns, two alternative downside risk measures were also utilised in addition to the conventional variance. These were Lower Partial Moment measures with target rates of zero and the individual assets mean return (the semi-variance). The results show that all three minimum risk portfolios out-performed the alternative Bayes-Stein and Classical portfolios on an ex-post basis. The results are also similar to the findings of papers such as Haugen & Baker (1991) in the analysis of individual securities. Haugen & Baker (1991) compared the ex-post performance of minimum variance portfolios against the US market, in an attempt to examine the relative performance of index funds. As the current study, like Haugen & Baker (1991) and Chopra, Hensel & Turner (1993), uses rolling portfolios a further advantage to the analysis of the minimum variance portfolios is that with the use of such short sample periods, the tangency portfolio by definition contains those asset classes, or securities, that have produced the best performance over the proceeding period. The strong ex-post performance is therefore consistent with the literature on mean reversion and contrarian strategies[6].

Data & Methodological Framework

A total of eleven markets are examined in this study, over a total sample period of 1976 to 1998[7]. All eleven markets are analysed using monthly data, with the Datastream property indices representing each of the markets with the exception of the USA, in which case the NAREIT index is used. An assumption is made that an investor cannot partake in short selling, due to the fact that most institutional investors are restricted in this regard. All of the data is analysed on the basis of local returns, thereby implying perfect hedging ability. While the use of such an assumption does ignore the impact of the foreign exchange market, it does mean that additional assumptions concerning the nationality of the investor are avoided[8]. Table 1 provides details of the summary statistics of the data for the overall sample period.

The study initially attempts to gauge the potential cost of estimation error of the mean, variance and covariance. Studies such as Kalberg & Ziemba (1984) and Chopra & Ziemba (1993) have found that the importance of error in the mean is substantially greater than the relative importance of errors in the variance and covariance. The methodology used to assess the relative importance of different forms of errors is similar to that used by Chopra & Ziemba (1993) and uses the overall data set of 276 observations. Assuming that the historical estimates for the parameters are the true figures, a base optimal portfolio is calculated that maximises the Sharpe Ratio. To assess the impact of estimation error in the mean, we replace the historical estimate $\bar{r}_i$ for asset *i* with $\bar{r}_i\left(1+kz_i\right)$, where *k* is allowed to vary between 0.05 to 0.30 to assess the impact of different magnitudes of errors and *z* has a standard normal distribution. Similar corrections are then performed with respect to the variance and covariance. In each case the remaining two parameters are left unaltered, while the procedure is completed 100 times for each value of *k* for a different set of *z* values. The mean absolute difference from the historical estimates are then calculated for each value of *k*.

The ex-post analysis is undertaken on the basis of a 60 month rolling window. The optimal portfolios are then re-calculated every quarter. Three alternative portfolio construction strategies are used. Initially the classical tangency portfolio is used, while the two alternatives are the Bayes-Stein approach, using the suitable prior proposed by Jorion (1985), and the minimum variance portfolio. As the minimum variance alternative does not use the means in the calculation of the allocations, the estimates are identical whether the original or ‘shrunk’ mean returns are used. Portfolios based on the three alternate strategies are then constructed and the performance of them is examined on an ex-post basis and compared to a naive equally weighted portfolio of the eleven markets.

Empirical Analysis

Initially the potential impact of variations in the inputted parameters is examined. Using the procedure described in the previous section, the mean absolute differences from the returns obtained using the sample data is presented in Table 2. It can be seen quite clearly that while the error associated with the two risk measures does generally increase with the value of *z* the impact remains relatively minimal. In contrast however, the impact of variations in the means is substantial. At the smallest value of z the impact of estimation error from the mean is greater than any of the values for either the variance or covariance, with the figure rising to 7.39% when z equals 0.30. The potential biases that can arise from sample means therefore, provides further justification for the use of the techniques used in this study.

Charts 1 through 3 show the rolling allocations in each of the eleven markets. While, the broad patterns are similar, it can be seen that the mean-variance tangency case has the highest degree of variation. The use of the Bayes-Stein shrinkage does reduce the degree of sudden changes in the allocations, a process that is continued by the use of the minimum variance portfolio. In that case the portfolio is dominated in the early period by the Dutch market, while the REIT market in the United States dominates the period from 1991 onwards. Due to the use of the 60 month rolling window, the portfolios are analysed over an 18 year period from January 1981 to the end of year 1998, with Table 3 providing the summary statistics of the alternative portfolios constructed, together with the equally weighted naive portfolio. Of the four alternatives the classical tangency approach produces the worst ex-post performance with a mean monthly return of 0.72% and a standard deviation of 3.77%. The Bayes-Stein prior portfolio not only obtains a higher ex-post mean return, but the risk of the portfolio is also reduced, with figures of 0.78% and 3.30% respectively. In addition, the holding period return increases from 27.19% to 38.01% over the eighteen year period. However, of the three approaches the minimum variance portfolios, which totally excludes the problem of estimation error resulting from bias in the means, provides a further improvement in performance, with additional increases in the return figures and reductions in the risk measures. If the results are compared against the naive strategy, it can be seen that while the equally weighted index provides a higher return than both the classical and Bayes-Stein tangency portfolios, it does result in increased risk measures. Using the Sharpe ratio as a further comparison of performance, it can be shown that both the Bayes-Stein and Minimum variance portfolios outperform the naive portfolio.

To more formally assess the ex-post performance of the alternative portfolios we use the Jobson & Korkie (1981) pairwise test of the equality of Sharpe Ratio. The test statistic can be displayed as:

$$t = \frac{s_j \bar{r}_i - s_i \bar{r}_j}{\left[2 / T\left(s_i^2 s_j^2 - s_i s_j s_{ij}\right)\right]^{1/2}} \tag{4}$$

where s_j is the standard deviation of asset *j*, $\bar{r}_j$ is the mean return of *j* and s_{ij} is the covariance between assets *i* and *j*. The results, reported in Table 4 reveal that both the Bayes-Stein and minimum variance portfolios significantly outperformed the mean-variance portfolio, with t-statistics significant at the 95% level. With regard to the naive strategy, the test results for the mean-variance and Bayes-Stein approaches were insignificant, therefore, while it cannot be shown that the shrinkage approach leads to out-performance against an equally weighted index, it also cannot be shown that the classical optimisation approach does not significantly under-perform. The results do however confirm the strong ex-post performance of the minimum risk portfolio, with this strategy providing significant out-performance against all three alternatives. It should be noted this test has low power, as noted by Jobson & Korkie (1981), therefore, the finding of any significant results is to some degree surprising[9].

Tables 5 and 6 more formally examine whether an investor significantly gains from investing in foreign markets. To assess this issue, the four portfolios are compared against the individual market returns over the ex-post period, 1981 to 1998. It can be seen that in comparison to the classical tangency case, six of the eleven individual markets produce higher average out-of-sample mean returns. Even with the adjusted optimal portfolios, and the equally weighted naive strategy, five of the markets produce higher returns. However, if the risk measures are compared, it can be seen that in the vast majority of cases the greatest benefit from diversifying internationally comes from the reduction of risk. In the cases of the Bayes-Stein and Minimum Variance portfolios, none of the individual markets have lower standard deviations. Even in the case of the unadjusted tangency portfolio, only the American and Dutch markets have lower risk measures. This is also the case with the equally weighted portfolio. The resulting lower risk measures, means that in the majority of cases the corresponding Sharpe ratios are lower for the individual markets.

We again use the Jobson & Korkie (1981) pairwise test of the equality of Sharpe Ratio to compare ex-post performance, the results being reported in Table 6. In each case, the international diversification strategy outperforms domestic portfolios for Canada and Japan. Therefore, in the case of these markets the perceived benefits from diversifying into international markets is further confirmed. In addition, the naive strategy produces significant out-performs the Italian and French markets. The two portfolios constructed to reduce estimation error provide further evidence as to their attractiveness. The Bayes-Stein prior portfolio significantly outperforms six of the eleven markets, while the minimum variance strategy sees significant out-performance in eight of the eleven cases. The only exceptions are in the case of Australia, Belgium and the United States. The only cases where a domestic market outperforms the international strategy, thereby implying no benefits to diversifying into foreign stocks, are with regard to Australia and Belgium for the original tangency portfolio, although in neither case is the test statistic significant. The REIT market in the US however, outperforms all four portfolios, and is statistically significant in the case f the mean-variance tangency portfolio. Therefore, these results would imply, that American investors in REITs gained no benefits from extending their portfolio into an international environment.

Concluding Comments

Much of the existing literature has ignored potential biases in a standard mean-variance approach. This paper has provided preliminary evidence as to the attractiveness of addressing the issue of estimation error in asset allocation studies. The problem of estimation error is not solely a theoretical one, as has been shown in this paper, as the use alternative techniques can lead to a reduction in the variation is the estimated portfolio allocations and can lead to improved ex-post performance. As with previous studies, the use of the Bayes-Stein shrinkage approach does lead to increased stability in the estimated allocations and results in improved ex-post performance. However, the greatest improvement in out of sample performance came from the use of the minimum variance portfolio. In this scenario, all estimation arising from the sample means is eliminated as the minimum variance portfolio does not use the means in the determination of the allocations. Not only does the MVP portfolio outperform the classical tangency portfolio and the Bayes-Stein estimated portfolio, but it significantly outperforms a naive equally weighted strategy.

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Chart 1: MV Maximum Sharpe Ratio Allocations

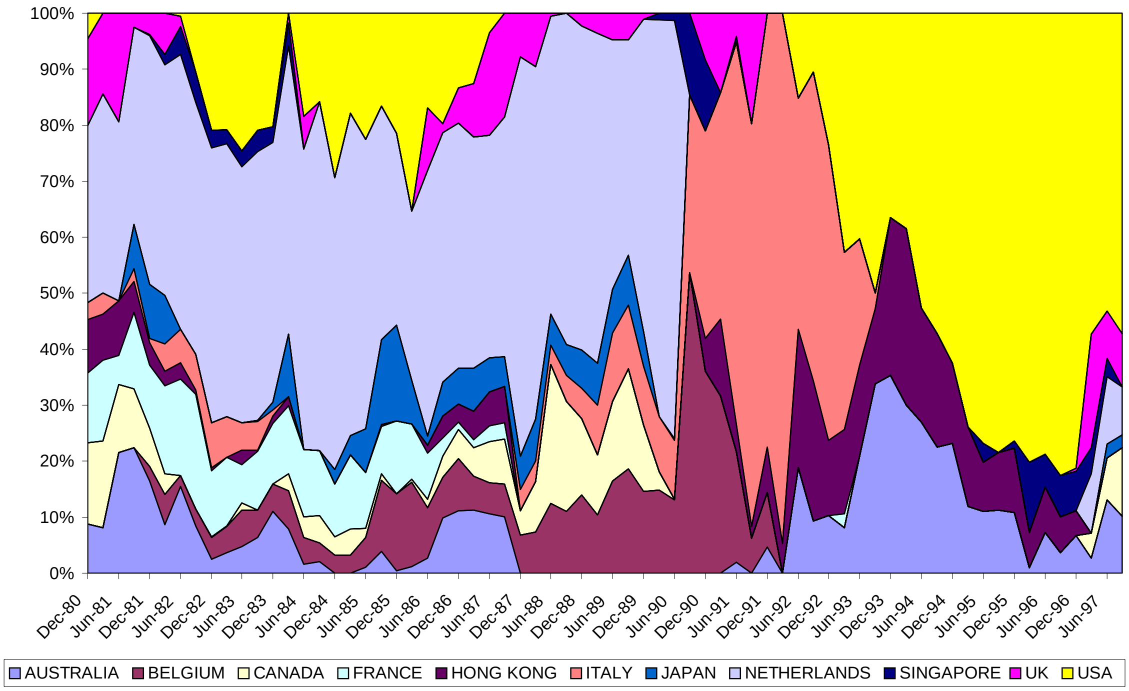

Chart 2: Bayes Stein Prior Maximum Sharpe Ratio Allocations

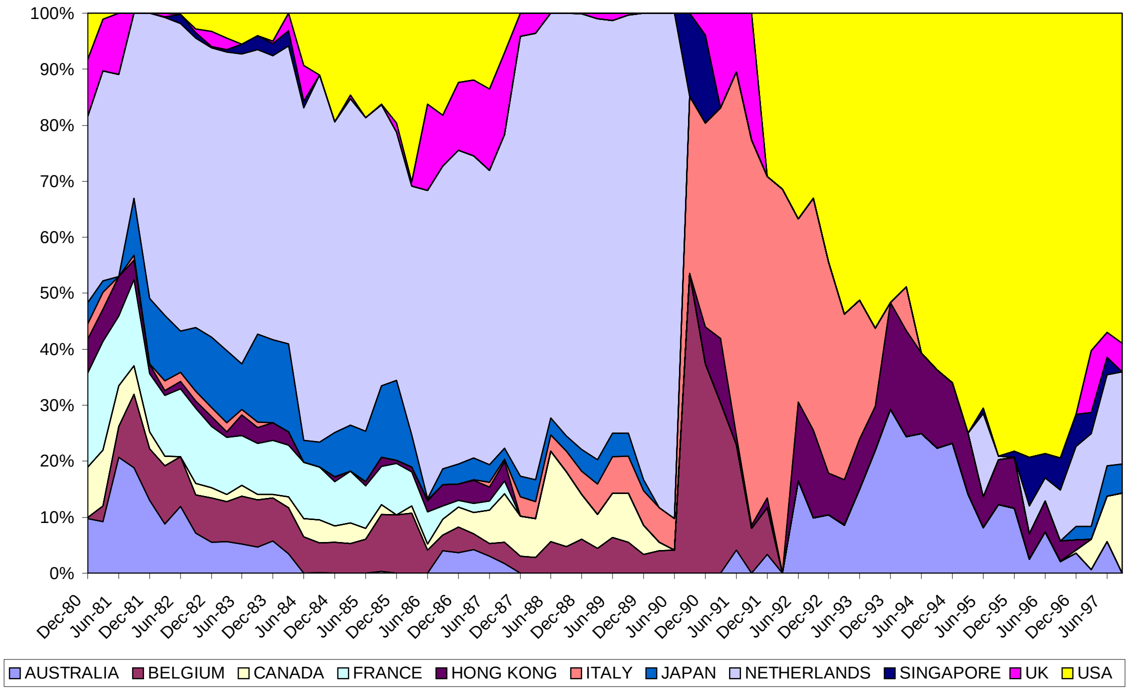

Chart 3: Mean Variance Portfolio Allocations

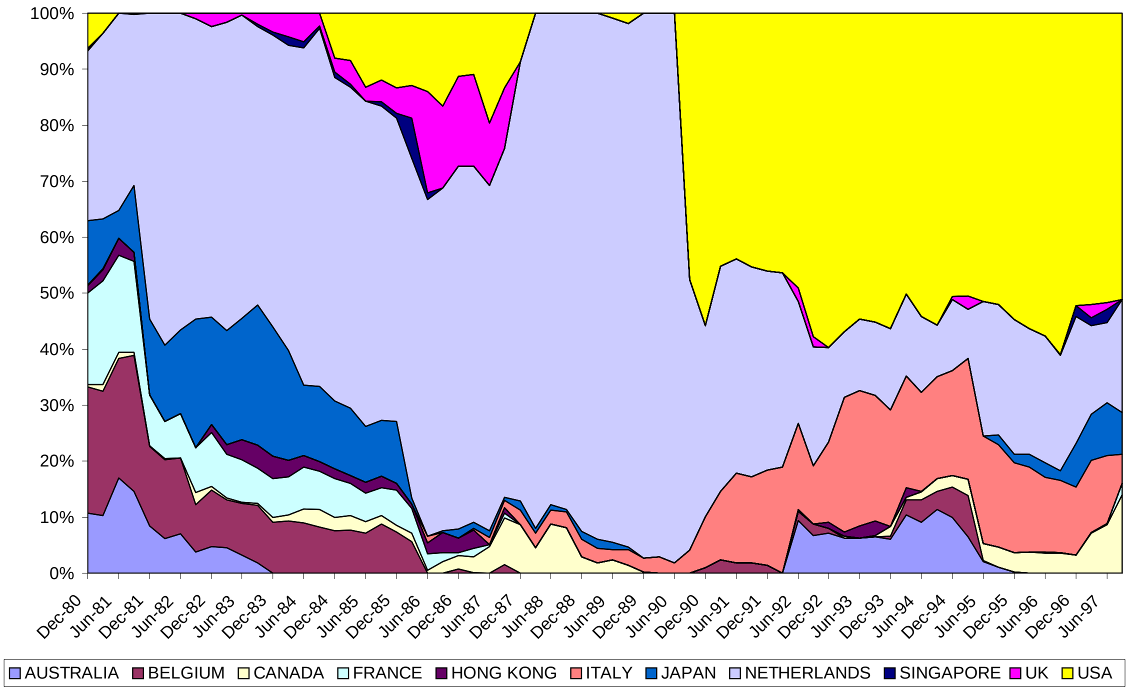

Table 1: Sample Statistics

	Mean	Standard Deviation	Variance
Australia	1.6186	7.5380	56.8222
Belgium	0.9126	6.2860	39.5134
Canada	1.0183	9.5618	91.4281
France	0.7878	8.4027	70.6058
Hong Kong	1.6565	11.7140	137.2178
Italy	1.0665	8.2164	67.5090
Japan	0.5549	8.2369	67.8473
Netherlands	0.6794	3.7730	14.2354
Singapore	1.0295	11.0161	121.3536
UK	1.2800	6.4805	41.9965
USA	1.1925	3.6209	13.1109

Table 2: Impact of Estimation Error

Z=	Means	Variances	Covariances
0.05	1.23%	0.12%	0.02%
0.10	2.46%	0.14%	0.07%
0.15	3.69%	0.08%	0.17%
0.20	4.92%	0.13%	0.27%
0.25	6.16%	0.18%	0.39%
0.30	7.39%	0.24%	0.51%

Table 3: Ex-post Performance

	Maximum Sharpe Portfolio	Bayes-Stein Maximum Sharpe Portfolio	Minimum Variance Portfolio	Naive Portfolio
Mean	0.7185	0.7750	0.8849	0.8684
Standard Deviation	3.7693	3.2993	2.9348	4.1662
Variance	14.2079	10.8851	8.6128	17.3570
Sharpe Ratio	0.1906	0.2349	0.3015	0.2084
Holding Period Return	27.1930	38.0079	53.5148	45.7356

Table 4: Comparison of Ex-post Performance

	Maximum Sharpe Portfolio	Bayes-Stein Maximum Sharpe Portfolio	Minimum Variance Portfolio
Bayes-Stein Maximum Sharpe Portfolio	-2.1904**		
Minimum Variance Portfolio	-2.1172**	-1.5596*	
Naive Portfolio	-0.3822	0.5083	1.5568*

Table 5: Individual Market Ex-post Performance

	Mean	Standard Deviation	Variance	Sharpe Ratio
Australia	1.6018	7.7160	59.5369	0.2076
Belgium	1.3292	6.5279	42.6138	0.2036
Canada	0.4414	9.5325	90.8679	0.0463
France	0.6858	9.0337	81.6072	0.0759
Hong Kong	1.1383	11.5181	132.6663	0.0988
Italy	0.5737	7.1939	51.7520	0.0797
Japan	0.5014	8.9206	79.5779	0.0562
Netherlands	0.5224	3.4650	12.0062	0.1508
Singapore	0.7294	11.0556	122.2255	0.0660
UK	0.9984	6.0276	36.3316	0.1656
USA	1.0304	3.2481	10.5499	0.3172

Table 6: Comparison of Ex-post Performance Between Optimal Portfolios and Individual Markets

	Maximum Sharpe Portfolio	Bayes-Stein Maximum Sharpe Portfolio	Minimum Variance Portfolio	Naive Portfolio
Australia	-0.1732	0.2786	0.9579	0.0086
Belgium	-0.1323	0.3191	0.9980	0.0491
Canada	1.4639*	1.9122**	2.5862***	1.6476**
France	1.1651	1.6158*	2.2944**	1.3475*
Hong Kong	0.9325	1.3817*	2.0550**	1.1143
Italy	1.1267	1.5777*	2.2570**	1.3078*
Japan	1.3637*	1.8150**	2.4940***	1.5460*
Netherlands	0.4095	0.8693	1.5788*	0.5917
Singapore	1.2655	1.7150*	2.3913***	1.4495
UK	0.2551	0.7077	1.3868*	0.4377
USA	-1.3095*	-0.8550	-0.1640	-1.1227

[1] The three markets that provided significant results when spot foreign exchange rates were used were Japan, the Netherlands and Singapore. The other markets to be examined in this study were Australia, Belgium, Canada, France, Italy, the UK and the United States.

[2] The paper used the methodology proposed by Gibbons, Ross & Shanken (1989)

[3] In addition to the standard MVA approach Liu & Mei (1998) also analysed the issue using a Multifactor Latent Variable Model.

[4] See Jorion (1985) for an extended discussion on this point.

[5] Further papers to find that ex-post performance is improved when estimation risk is accounted for include Eun & Resnick (1988) and Stevenson (1999),

[6] See for example Fama & French (1988), Poterba & Summers (1988) and Lakonishok, Shleifer & Vishny (1994) with respect to the evidence concerning individual stocks. In addition, papers such as Richards (1997) and Balvers, Wu & Gilliland (1999), provide evidence of mean reversion in national stock indices.

[7] The countries analysed are Australia, Belgium, Canada, France, Hong Kong, Italy, Japan, the Netherlands, Singapore, the UK and the USA.

[8] Stevenson (2000a) analysed the diversification opportunities from extending into international markets from the perspective of each of the countries examined. The study found that substantial differences can occur in the results between the assumed nationality of the investor when currency movements are taken into account. This is one aspect of the current study that is to be extended.

[9] A shortcoming of the analysis contained in this paper is that at present it does not incorporate transaction costs into the analysis. The author is currently extending the analysis to include this aspect, therefore, the present results should be viewed in light of this omission.