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Can a Hybrid Automated Valuation Model Outperform Individually Assessed Capital and Site Values.

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Abstract: In recent years the use of Hybrid Automated Valuation Models has been widely discussed in the property taxation literature. Such models are now recognised in the IAAO's standard for AVM's. Given recent court decisions in Australia that seem to require valuers to consider sales with improvements when assessing site value in "thin markets", such models may prove to be a useful tool in mass appraisal. This is particularly relevant following problems with the delivery of acceptable valuations for rating and taxation in several states in Australia. In 2004 a variety of AVM's were developed for a study area in Adelaide and presented at the 2005 PRRES conference. That research showed that a single model could produce capital and site values of a similar average accuracy to the actual assessed values that were created using multiple computer assisted valuation (CAV) models at the small sub-market level. This paper extends the research, by improving the models using the outcomes from the previous research but also by applying a wide range of tests. Assessments based on a large number of small models may lead to assessment bias and an abnormal distribution of assessment ratios typically evidenced by low relative valuations for high priced properties. In this research various measures from the IAAO standards on ratio studies are adopted as the tools to assess the performance. In particular the A/S ratios are tested for the level of assessment using mean, median, weighted mean and geometric mean and for variability using the coefficient of dispersion (COD),coefficient of variation (COD) and quartile ranges whilst reliability is tested using confidence intervals and vertical inequities with the price related differential (PRD) and normality with the Shapiro-Wilk test. The Mann-Whitney test is used to check for sales chasing and reliability between the model and hold-out data.

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Introduction

In recent years the use of hybrid Automated Valuation Models (AVM's) has been widely discussed in the property taxation literature as well as in many individual reports where the results of such models are used in jurisdictions for mass appraisals. Such models are now recognised in the International Association of Assessing Officers (IAAO) standard for AVM's. These models may produce superior results to simpler AVM's and may be particularly useful in a situations where improved properties need to be used to estimate site value. Given recent court decisions in Australia that seem to require valuers to consider sales with improvements when assessing site value in "thin markets", such models may prove to be a useful tool in mass appraisal. This paper examines the use of a hybrid model to estimate capital and site values for residential properties in a small pilot study in Adelaide, South Australia. The model uses both vacant land and improved residential sales in a single model to estimate both site and capital values. The issue of mass appraisal methods is topical following recent problems in various Australian States with the delivery of assessments for rating and taxation. The most recent of these is in New South Wales (NSW Ombudsman, 2005) following an enquiry into concerns about the consistently of assessments. The report states that

> "Valuers increasingly rely upon sales of improved properties rather than vacant land sales due to the scarcity of the latter and a recent court precedent. Currently there is an absence of a uniform methodology for valuing improvements"

Models that jointly assess improved and site values may prove to be a superior means of estimating the inherent value of the improvements as well as offering a fast and efficient method of assessment for both site and capital values.

Hybrid Automated Valuation Models

The IAAO standards define an automated valuation model (AVM) as

> "… a mathematically based computer software program that produces an estimate of market value based on market analysis of location, market conditions, and real estate characteristics from information that was previously and separately collected. The distinguishing feature of an AVM is that it is a market appraisal produced through mathematical modelling. Credibility of an AVM is dependent on the data used and the skills of the modeller producing the AVM" (IAAO, 2003 pp148)

They further recognize that these may be in an additive, multiplicative or hybrid form where the hybrid form is a

> "… model that incorporates both additive and multiplicative components" (IAAO, 2003 pp150)

and that these are normally hedonic models which attempt

> "… to take observations on the overall good or service and obtain implicit prices for the goods and services. Prices are measured in terms of quantity and quality. When valuing real property, the spatial attributes and property specific attributes are valued in a single model. Calibration of the attribute components is performed statistically by regressing the overall price onto the characteristics." (IAAO, 2003 pp149)

In a study researching the valuation of land and improvements in the City of Philadelphia, McCain, Jensen et al. (2003) use some 40,000 arm's length transactions to develop a two stage hybrid model. The first stage involved estimating a neighbourhood index for each property which was then used as input to a hybrid regression model. The neighbourhood index was estimated from the residuals of a simple hedonic model (using building and site characteristics) and then used a Kriging process to smooth out the variation.

This neighbourhood variable was then combined with land area, liveable area and building condition in a non-linear regression. The hybrid model was specified as (sic)

$$p = (b0(LiveableArea)^{b1}(Condition)^{b2}(Neighborhood)^{b3}) + (b4(LandArea)^{b5}(Neighborhood)^{b6})$$

Where *p* is the price of the property.

This model is applied using both improved and unimproved sales and allows for the neighbourhood influence to be attached to both the land and improvements components at a different rate. Values for improved properties use the whole equation while vacant land estimates effectively use only the second component (since liveable area and condition are zero). This model proved to be effective even with a small set of descriptive variables.

McCluskey, Deddis et al. (1998) discuss various methods of building spatial variation into mass appraisal. They discuss the problems of using submarket analysis where the submarkets often become small and the statistical analysis becomes unsound and biased (they do not discuss this in terms of non-statistical methods but the same problem applies). They then discuss the problems of using dummy variables for discrete locations such as suburbs. They point out that this "presupposes that the affect of location is uniform across all properties within a particular neighbourhood". This method also causes problems for mass appraisal authorities because of the lumpiness of assessments and border conflicts. They suggest that a more continuous approach using methods such as surface response analysis and the kriging method. These methods may be applied through several of the standard GIS packages.

A study of three alternative models (additive, multiplicative and non-linear) was reported by O'Connor (2002) based on work in Calgary. They used a large geographical area and used some 35,000 records randomly split into about $4/5^{th}$ for model building and $1/5^{th}$ for testing. They used a two level cleaning process each involving the removal of the lowest and highest 2.5% of estimate to sale ratios. They use two methods to allow for location influences; a location value response surface (LVRS) based on median prices and one based on fixed neighbourhood boundaries. Models are generated for each of the three model types and using both locational methods. The results are compared using the coefficient of dispersion (COD), coefficient of variation (COV) and price-related differential (PRD) as specified by the IAAO Standards on Ratio Studies (1999). They found a multiplicative model with LVRS to be superior for both the within model and the hold out data with a COV of 7 and 7.91 respectively.

In a similar study involving Calgary, Gloudemans (2002, see also Gloudemans, 2002a) followed similar procedures but used more discriminating sales selection based on transaction characteristics as well as high AS ratios. They split the data into testing and model build subgroups of 5000 and 25,303 sales respectively using random selection. They then created additive (linear), multiplicative (log-linear) and Hybrid (non-linear) models. Location was included in the model via a large number (hundreds) of neighbourhood dummies. The non-linear model is specified in a similar manner to that of McCain, Jensen et al. (2003) with the site and building parts being multiplicative and added together. They concluded that all three models produced good results but the multiplicative model produced the best results although they felt that it might not have produced the best results across the whole city and that the non-linear specification most closely fitted the appraisal theory.

Estimating Site and Capital Values

One important advantage of these hybrid models is that they may offer a suitable solution to the valuation of land for site value purposes in situations where the number of sales is low, generally called a "thin market". In the Maurici Case (High Court of Australia, February 13, 2003) summarised by Collins (2003) and applauded by Robbins (2003), the valuer was criticised for failing to consider improved properties when estimating the unimproved value and relying upon a small number of sales from a very thin market. The relative judgments in this case will not be debated in this paper but the case has re-opened the debate about using such traditional methods as the cost approach. While it is clear some of the writers on this issue have a fundamental misunderstanding of market valuation and a naive understanding of the use of cost to estimate value, the opportunity to reopen the analysis of improved sales to value vacant sites in a quantitative manner should be welcomed by academics and practitioners. One particular use of such analysis is in the derivation of site values for rating and taxation purposes.

In Australia the basis for valuation for rating and taxation purposes varies from state to state. New South Wales and Queensland use unimproved value; Victorian councils have a choice of assessing capital value, net annual value or site value; Tasmania assesses capital improved value, land value and assessed annual value; Western Australia assess gross rental value, site value (urban), unimproved value (rural) and capital value (government owned properties) and South Australia assesses both capital value and site value for every property. Generally, site or unimproved value is used for land tax while the other bases may be used for other purposes. Site or unimproved value is assessed in all jurisdictions but is fraught with difficulties in many of the established urban (and rural) areas due to the low number of market transactions. While unimproved value is a hypothetical and non-market testable construct in most cases, its foundation is in the market for vacant rather than improved sales. If the findings of Maurici are accepted as reasonable then the scarce sales of vacant land may not be sufficient to indicate the true market value of vacant land (and therefore unimproved land) and transactions of improved properties should also be considered. Since the cost of construction rarely equals the added value of improvements (the added value tends to be either above or below the cost of construction depending on the relative supply-demand situation) this is not a suitable method for “splitting” improved sales prices into a land and building component. But this may be possible using market analysis that jointly considers the sales of both improved and vacant properties. A properly calibrated hybrid model may meet these demands. In South Australia where every property must be assessed for both capital and site value on an annual basis, such a hybrid model may serve the purpose of completing all valuations from a single model and lead to acceptable estimates of both site and capital value. The results are also useful for “component” assessment adjustments, such as an increase in assessment for new additions. It is with these aims in view that this research has been carried out.

Evaluating the accuracy of the models

Evaluations of AVM’s is generally conducted using the IAAO standard on Ratio Studies (IAAO, 1999) which is currently being reviewed (IAAO, 2005). The standard is based on the comparison of assessed values to market sale prices or independent valuations. Typically it is used to test assessed values against market sales data during the assessment creation process and then to tests the assessment against sales that occur after the assessments have been finalised. In this way it is useful to determine if the assessment is likely to be accurate prior to release and then as a tracking mechanism to test for actual accuracy after the assessment is released. Ratio studies rely upon the use of the A/S ratio; the ratio of assessed value to the sale price (or independent valuation). The A/S ratios are then charted in various ways, described and inferential statistics used to determine the accuracy of the assessments. Typically the study uses a variety of parametric and non-parametric tests. These tests include:

1. Measures of assessment level; mean; median; weighted mean and geometric mean ratios.
2. Measures of Variability; Coefficient of Dispersion (COD); Coefficient of Variation (COV) and quartile ranges.
3. Measures of Reliability; confidence intervals and standard errors.
4. Vertical Inequities; Price Related Differential (PRD)
5. Hypothesis tests; Normality (for example Shapiro-Wilk test), two groups tests for equality (Mann-Whitney), three or more groups tests for equality (Kruskal-Wallis), sales chasing (Mann-Whitney).

To facilitate the number of tests used in this paper an Excel add-in package was developed to enable the fast and consistent calculation of many of these statistics for each group of assessments.

Methodology

Study Area

This study is conducted in a small section of Metropolitan Adelaide incorporating nine suburbs. The area is located in the southern suburbs (see Appendix Figure 1) wedged between the sea to the west, hills to the east, a river and commercial district to the south and an industrial area to the north. The location contains a mixture of housing established over a 40 year period in a number of expanding developments. As a result some parts of the study area have predominantly improved sales and few vacant land sales while the newer locations have larger volumes of vacant land sales.

Study Period

The study is completed using data from 1998 and 1999. This period was chosen for three reasons. It reflects a period of time when the residential property market in the area was relatively stable and therefore time adjustments are not necessary within the models. It is also a period when the quality of data is considered to be superior. In recent years there has been a concern that some property characteristic data held by the government (and made available to industry and research groups) has become less reliable as funding for appropriate staff is reduced. This data period is more likely to have a better quality of data. Thirdly this period was used in a pervious study of Adelaide that included results for parts of this location (Rossini, 1998) and provides valuable additional information.

As in previous studies the data was broken into two groups. The first group would be used to create models and the second group to test the models. This is a standard holdout sample procedure typical of most forecasting and prediction methodologies and is designed to prevent overestimating the accuracy of the models where over-fitting occurs. For this study, designation of these two data sets was based on a logical rather then random approach. If the model were to be used to assess capital and site values then the normal procedure would be to use sales from one period to estimate the values for the forthcoming assessment period. In this study we assume that the task is to create capital and site assessments in 1999 using the data from the previous year

(1998) and that the assessments are then evaluated at the end of 1999 using the sales that occur during the 1999 period as the accuracy test. While this is likely to cause some “on-average” under-assessments if prices have been increasing, it does create a more realistic model and test situation.

Data

For this study only detached houses and vacant land are used and allotment sizes are limited to those between 200 and 2000 sq metres. This would include the vast majority of all land uses in the study area. A large amount of data is available for each property but many of these are descriptors (such as the title reference) are not used in AVM’s and some other variables are not collected for every property. One significant set of data not available on the sales history file is a geographic midpoint for each property (which might typically be generated from a GIS). These were added to the data set from a matched file of latitudes and longitudes. The variables listed in Table 1 are suitable for the use in the AVMs and were available for every property with the building characteristics being zero in the case of vacant properties.

Table 1 - Variables used in the AVM's

Variable	Variable Name/Description
Sale Price	SalePrice
Sale Date	SaleDate
Longitude	Easting and converted to a simple grid reference (X)
Latitude	Northing and converted to a simple grid reference (Y)
Land Area	Larea
Building Area	Barea
Number of Main Rooms	Rooms
Building Condition Code	Condition
Building Age	Bage
Outer Wall cladding	Converted to Dummy Variables
Roof cladding	Converted to Dummy Variables
Building Style	Converted to Dummy Variables

All relevant transactions were extracted from the sale history file and cleaned for observations with missing data or where the price was demonstrably incorrect. Further sales were deleted that had A/S ratios that were outside of the plus or minus 2.5 standard deviation range in terms of improved properties and 3 standard deviations for vacant properties. Unlike previous studies, properties that did not accurately model were not excluded. All data removed was on an a-priori basis rather the ex-post approach taken by both O’Conner and Gloudemans where properties that are poorly estimated in the models are removed. That approach will tend to overestimate the accuracy of the models as some of these will be properties that are genuine transactions with correct data but that the model is incapable of properly estimating. The likely cause being omitted variables. By removing such data the opportunities to investigate these omitted variables is lost and the accuracy of the model appears better both in terms of the model statistics and the test statistics where difficult to assess properties have been removed. The approach taken in this study is to remove only those observations that are clearly incorrect or where there is missing data making it impossible to use the observations. This means that the estimates of model accuracy become quite conservative and would only be improved by diligent sales analysis and data rechecking. These would normally be carried out by a rating authority in the process of mass appraisal. As a result it is likely that a number of gross outliers will appear in the test assessments that would not occur in a true mass appraisal.

Following this basic cleaning process there were 2367 observations. A break down of these observations by suburbs and land use is shown in Table 9.

Modelling

The following is a summary of the process was used in the multi-stage modelling. All regression models are estimated through an OLS or GLS process in SPSS.

- Step 1. Split the data into model and test data
- Step 2. Use the model data to develop linear and log-linear AVM's using the building and site characteristics. Select the best model and save the standardised residuals.
- Step 3. Using the standardised residuals from step 2 use the latitude and longitude as and X and Y coordinate and polynomial expansions of these variables to establish a location value response surface (LVRS).
- Step 4. Using the surface in step 3 – estimate a new spatial variable (LOCATION) for all observations. This should account for special major effects primarily in the land component and is a surrogate for a variety of spatial characteristics that might be collected through a GIS.
- Step 5. Develop linear, log-linear and non-linear (hybrid) models using the location, site and building characteristics.
- Step 6. Estimate the value (assessments) for all properties using each of the three models developed at step 5.
- Step 7. Calculate A/S ratios and associated statistics for model and test data and for both vacant and improved properties for assessments from step 6 based on the IAAO standard for ratio studies (IAAO 1999).

Results

As the models are developed over a staged process, model specifications and estimates are shown for the various stages.

Step 1 – Data splitting

The data is split into 4 data types. The frequency of these data types is shown in Table 2.

Table 2 - Summary of data by type and year

TYPE	Year of Sale	Frequency	Percent
Dwelling - Model	1998	917	38.7%
Dwelling - Test	1999	1081	45.7%
Vacant - Model	1998	186	7.9%
Vacant - Test	1999	183	7.7%
Total		2367	100.0%

Step 2 - Initial models to find residuals for use in the LVRS

Basic linear and non-linear regression models were established to find systematic error to model in the LVRS. For this model only 1998 sales (dwelling-model and vacant-model) were used. After some experimentation a linear model proved to be most reliable. This model is specified as

$$\hat{P} = b_0 + b_1 X_1 ... b_n X_n + (\theta + \varepsilon)$$

Where

P	=	transaction price
b_0	=	a constant
$b_{1...}b_n$	=	market determined parameters
$X_{1...}X_n$	=	a vector of property characteristics
θ	=	systematic spatial component captured in the residual
ε	=	stochastic errors included in the residual

Estimates and statistics for this model are shown in Table 10. The systematic spatial component and stochastic error are captured in the residuals from this model. These residuals (in standardized form) become the inputs to the model to establish the LVRS.

Step 3 - Estimating the land value response surface (LVRS)

A number of methods are available to find an LVRS. Using the same data set, Rossini et al (2004) found that methods such as kriging tended to over fit the surface while a regression using polynomial expansions of the latitude and longitude variables, produced a simple but logic surface in this location. The model is estimated using regression (OLS) with a cubic polynomial expansion of the coordinates and is specified as:

$$\hat{\theta} = b_0 + b_1 X + b_2 Y + b_3 X^2 + b_4 Y^2 + b_5 XY + b_6 Y^2 X + b_7 X^2 Y + b_8 X^3 + b_9 Y^3 + \varepsilon$$

Where

θ	=	Systematic spatial component (standardized residuals)
b_0	=	a constant
$b_{1...}b_n$	=	market determined parameters
X		Easting and converted to a simple grid reference (X)
Y		Northing and converted to a simple grid reference (Y)
ε	=	stochastic errors

Estimates and statistics for this model are shown in Table 11 and a graphical representation of the value surface is shown in Figure 2. The surface shows the expected responses with higher values along the coast line to the west and along the elevated hills area to the east. Values in the central area and near the industrial estate are lower with the lowest value being associated with a newer estate located near the commercial-shopping area to the south. The model shows an R squared value of .194 which is significant at a greater than 99% level of confidence.

Step 4 – Estimate the location variable to allow for spatial effects

The model from step 3 was used to estimate the new location variable for every property in the data base using the properties relative position on the location value response surface. This variable was added to the data set and the models re-estimated with the inclusion of the location variable.

Step 5 - Develop linear, log-linear and non-linear (hybrid) models

Three different models are developed for use in the assessment generation. While it is expected that the hybrid model will produced superior results, a linear and log-linear model are also developed for comparison. In an earlier study in this area, Rossini et al. (2004) discovered that a hybrid model produced slightly better result on an average error basis (MAPE). In this paper these three models are compared using a wider range of ratio statistics. The models are specified as follows

Linear

$$\hat{P} = b_0 + b_1\theta + b_2 X_1 ... b_n X_n + \varepsilon$$

Log-Linear

$$\hat{P} = e^{b_0 + b_1\theta + b_2 X_1 .... b_n X_n} \varepsilon$$

Where

P	=	transaction price
b_0	=	a constant
$b_1 ... b_n$	=	market determined parameters
θ	=	a vector of spatial location factors (step 4)
$X_1 ... X_n$	=	a vector of property characteristics including variables in ln and X^2 form
ε	=	stochastic errors

These models are estimated using a stepwise approach with manual manipulations to prevent multicollinearity becoming an issue. The choice of variable form (ordinary, logged or squared) is based on model performance.

Non-Linear (Hybrid)

$$\hat{P} = (b_0 \theta^{b1} Larea^{\,b2}) + (b_4 BArea^{\,b5})(Bage^{\,b6} Condition^{\,b7})(b_8{}^{D1} .... b_n{}^{Dn}) + \varepsilon$$

Where

P	=	transaction price
$b_0 ... b_n$	=	market determined parameters
θ	=	a vector of spatial location factors (step 4)
Larea	=	a vector of land areas
BArea	=	a vector of building areas
Bage	=	a vector of building ages
Condition		a vector of building condition codes
$D_1 ... D_n$	=	a vector of property characteristics as dummy variables
ε	=	stochastic errors

This model is specified exactly and is based on the finding of research in the same study area (Rossini, 2004). Since the non-linear model uses a generalised least squares approach (as opposed to ordinary least squares) that is based on an iterative approach, it is necessary to provide starting estimates for all the model parameters (regression coefficients). These starting values were estimated from two preliminary regression models firstly using vacant land sales and then using the improved sales. This followed the procedure taken by McCain, Jensen et al. (2003).

Model estimates

The results of the various models are shown in the appendix. The coefficients and statistics for the linear model are shown in Table 12, the log-linear model in Table 13 and the non-linear (hybrid) model in Table 14. The linear model is superior to the earlier model without the location variable (Table 11). The later model has an r-squared of .869, lower standard error of the estimate and much higher F ratio both suggesting it will provide better estimates. The second unrestricted model is statistically superior to the original model with the restriction (excluding the location variable). The log-linear model produces a higher F ratio and R squared but this is not sufficient to conclude that it is a statistically superior model. The hybrid (non-linear) model is probably superior with a corrected r-squared of .885. The lower residual sum of squares error suggests it is also superior. In all three models the coefficients are of the correct sign with high degrees of significance. Each model should produce adequate assessments.

Step 6 – Estimate the value (assessments) for all observations

The three models from part 5 are used to make price predations. These will be used as the new assessed values. Improved properties will be used to assess capital values and the vacant properties will be used to assess site values.

Step 7 - Evaluating the accuracy of the models

While the statistical analysis of the models is useful, the best test of a model's performance is the evaluation of assessments in the following year. Separate evaluations were carried out for the "model" (those observations with a 1998 transaction date that were used in estimating the model) and "test" data. (those observations that sold in 1999 which were not used in the creation of the model).

The measurement of assessment performance of the various models was carried out using AuditIT© which is a prototype add-in package for Microsoft Excel that is currently undergoing beta testing. Given an output dataset from a model or models that include the assessed value, the actual sale price and the date of sale, a collection of assessment-ratios are produced and tabulated. For this study the output is illustrated in Table 15 (linear and log-linear models) and Table 16 (hybrid and actual assessed values).

The assessment ratio is the assessed value divided by the sale price. Four measures of central tendency of the assessment ratio are calculated, the mean, median, geometric mean and the weighted mean. Dispersion is described using standard deviation, inter-quartile range, minimum, maximum and range. In addition the Coefficient of Dispersion (COD), Coefficient of Variation (COV) and Price Related Differential (PRD) are calculated.

$$COD = \frac{100}{\overline{AS}}\left(\frac{\sum_{i=1}^{n}\left|A_i/S_i - \overline{AS}\right|}{n-1}\right)$$

$$COV = \frac{100}{\overline{AS}} \sqrt{\left(\frac{\sum_{i=1}^{n} \left(A_i / S_i - \overline{AS} \right)^2}{n-1} \right)}$$

$$PRD = \frac{\overline{AS}}{\sum_{i=1}^{n} A_i \Big/ \sum_{i=1}^{n} S_i}$$

Where

$\overline{AS}$	=	mean assessment
n	=	number of ratios
A_i	=	assessment for property i
S_i	=	Sale price for property i

The mean, weighted mean and COV are sensitive to outlier ratios. Prior to calculation of the assessment ratios pruning of outliers is undertaken. The method adopted is that recommended by IAAO (IAAO, Draft 12 2005 pp28) where an "outlier" is defined as being outside of 1.5 times the inter-quartile range. This definition is based upon the work of John Tukey (1977) and the idea of boxplots. AuditIT© uses a convergence heuristic to prune outliers prior to calculating the assessment ratios.

Measures of assessment level

The mean and median ratios are measures of the assessment level and show that in 1998 the Valuer General set capital values for both improved and unimproved properties at about 93% of the market value. Most assessment authorities will set the mean and median slightly below 1 to allow for differences in definition between the assessment value and market value and to assist with a lower objection rate. Setting the level at 1 would suggest that approximately 50% of properties are assessed at or above market value. In South Australia the definitions used for capital value excludes fittings and fixtures and ornamental trees. This would typically mean that on average the assessments should be at around 90% to 95% of actual market value. In his paper discussing the accuracy requirements of automated and intelligent systems, Rossini (1999) analyses the accuracy of assessed values for detached dwellings over the whole Adelaide metropolitan area for sales within a 3 month period in 1998. He found a mean error of -8.48% suggesting a systematic underestimation of values of a little over 8%. The assessed values seem to be consistent with this. The models developed for testing should have a mean and median close to 1 as they should be unbiased estimators of transaction prices. The median A/S ratios for all three models are all between .985 and 1.02. Median assessment ratios for all models and for the actual assessed values were lower for the holdout or test data. The sales that occurred in 1999 show that typically the assessments were under-assessed by about 5%. For example the median for the actual assessed values dropped from .925 to .880 for the dwelling properties and from .933 to .838 for the vacant sites. The medians for the hybrid model dropped from .985 to .931 for the dwellings and from 1.00 to .962 for the vacant sites. The drop in the assessed capital values for improved properties is consisted with all three models and provides clear evidence that sales price increased by around 5% resulting in property being undervalued by about this amount. The 10% difference between the assessed values of vacant land in the model and test data is more concerning and may indicated some level of sales chasing or difficulties with setting the assessed values in 1998. This issue is discussed later. Overall the assessment levels are similar for all three models when considering the test data but higher (as expected) than the assessed values. Table 3 summarises the results for the test data.

Table 3 - Median A/S ratio comparison

Assessments	Dwellings	Vacant
Linear (Test)	.949	.984
Log-Linear (Test)	.949	.974
Hybrid (Test)	.931	.962
Actual Assessed (Test)	.880	.838

Measures of reliability and variability

The standard deviation is a useful measure of variability when assessment levels are the same. The COD and COV are more useful when assessment levels vary. The COD measures the average percentage deviation of the assessment ratios from the median and requires no assumptions about the shape of the population distribution. The COD is used as a measure of assessment uniformity to ascertain whether or not two or more sub-markets are assessed at similar percentages of market value. Acceptable levels of CODs for residential properties fall in the range of 5% to 15% and in homogenous neighbourhoods the COD should be at the lower end of this range.

The coefficient of variation only provides an appropriate measure of assessment ratio dispersion if their distribution is approximately normal. In such circumstances COV provides a complete portrayal of variability.

The COD and COV provide the most insightful analysis of the different models.

The actual assessed values are superior to all the models when considering the model data and both improved and vacant properties. The results also indicate that for improved properties the assessed values were still very accurate in the test data (1999 holdout sample). The COD changes from 7.3 to 8.1 and the COV from 9 to 10.1. Both are well within tolerance levels and show that the assessment made using 1998 data hold up very well against 1999 sales. The results based on vacant sales are however very different. The COD at 6.8 and COV of 8.6 are remarkably low for the model data but collapse to 18.5 and 22.7 respectively when considering the test (holdout) data. Clearly the assessments made in 1998 do not hold up against 1999 sales. While this may be due to sales chasing (changing the assessed values of properties that have sold to improve the statistics) the more probable reason is a high proportion sales that occurred in 1998 being in new developments. As a result the assessments would normally be set at their sale price when first sold. A smaller proportion of new developments in the following year will reduce the effect. This gives the appearance of sales chasing but is actually unavoidable. Noticeably when considering only the test data, the COD and COV for the actual assessments are very similar to the results for both the linear and log-linear models. Neither the linear nor the log-linear models were able to provide reliable assessments for vacant land (and hence site values) and this matched the poor reliability of the actual assessed values.

Table 4 - COD comparisons

Assessments	Dwellings	Vacant
Linear (Test)	9.6	17.8
Log-Linear (Test)	9.6	19.5
Hybrid (Test)	9.3	11.4
Actual Assessed (Test)	8.1	18.5

Table 4 summarizes the COD results. The Hybrid model is inferior to the actual assessed values for dwellings but is superior to the linear and log-linear models and within acceptable; tolerance. However in terms of vacant sites and hence site values, the hybrid model produces clearly superior results to the other two models and to the actual assessments. The hybrid model produces the only site value assessments that are within an acceptable tolerance.

Measures of vertical inequities

The price related differential provides a measure for the uniformity of assessment between low and high priced properties or vertical inequity. The PRD is centred about unity and where a PRD of less than 1 indicates progressivity and above 1 assessment regressivity.

Table 5 - PRD comparisons

Table 6 - PRD comparisons

Assessments	Dwellings	Vacant
Linear (Test)	1.015	1.054
Log-Linear (Test)	1.015	1.065
Hybrid (Test)	1.014	1.027
Actual Assessed (Test)	1.015	1.033

All the PRD'd show some degree of regressivity in the assessments. The results for the dwellings, (reflective of capital values) show that all assessment groups are slightly regressive but within normal tolerance. Although there are no official tests for this, most rating authorities will accept a range in the .98 to 1.03 range. When considering the evidence from the vacant land sales the actual assessed values are at the outer limit of the tolerance while the linear and log-linear models are unacceptably high. This suggests that in each of these instances, high priced property will be undervalued. The implication for site values is that all high valued properties will have lower proportional site values. Only the Hybrid model produces results that are acceptably within the tolerance for both the capital and site values.

Measures of normality

The "impossible dream" of all automated valuation models is to achieve complete accuracy and, thereby, uniformity of assessments. In this context a ratio analysis produces a COV of 0, a COD of 0, and a PRD of 1. Accuracy is reflected in the relationship between actual sale prices and the assessed values (after allowing for statutory stipulated differentials) and uniformity is determined by the consistency of assessment between high and low priced properties and between differing sub-market groups. In an imperfect market, using imperfect models, it is therefore necessary to fully analyse the ratios from the studies to determine whether they comply with the requirements of accuracy and consistency across sub-markets. The standard approach is to ascertain the distribution of the ratios; if they appear to conform to a normal or some other recognisable distribution then general parametric testing may be performed otherwise non-parametric approaches need to be adopted. The general assumption may be made that the majority of distributions will be non-normal (Utah, 2004).

In this study the Shapiro-Wilk test was used to test for normality. The results of these tests are shown in Table 7.

Table 7 - Normality Tests for Hybrid Model and Actual Assessed Values

Test of Normality Shapiro-Wilk		Statistic	df	Sig.
Actual Assessed	Dwell - Model	0.84884	900	0.0000
	Dwell - Test	0.84900	1049	0.0000
	Vac - Model	0.98381	179	0.0362
	Vac - Test	0.98043	161	0.0222
Hybrid Model	Dwell - Model	0.88633	879	0.0000
	Dwell - Test	0.87910	1059	0.0000
	Vac - Model	0.98916	173	0.2087
	Vac - Test	0.97681	168	0.0065

The Shapiro-Wilk test statistic is analogous to the correlation between the actual distribution and a normal distribution. A value of 1 indicates that the data is perfectly normal. For the actual assessments and the hybrid models, only assessments from the hybrid model of vacant land used to create the model, are statistically normal. Accordingly the nonparametric Mann-Whitney test was utilised to ascertain whether or not the level of assessment between paired models was similar and the Kruskal-Wallis test is an appropriate nonparametric test to use when seeking to compare three or more sub-markets. Table 8 shows the Mann-Whitney test for difference between the assessments in the model data and the test data for site and capital values for the hybrid model and the actual assessed values. In each case the results show that the level of assessment between the model and the test data are statistically significant. This supports the earlier results that show that there are different levels of assessment in 1999 compared to 1998. This is either evidence of sales chasing or a movement in the market. All of the evidence supports the proposition that this is due to a market movement as it appears in the three models as well as the actual assessed values.

Table 8 - Mann-Whitney Tests for Hybrid Model and Actual Assessed Values

Mann-Whitney Test of Unpaired groups		Mann-Whitney U	Sig.
Actual Assessed	Dwell - Model and Dwell - Test	471923	0.9917
	Vac - Model & Vac - Test	13181	0.1739
Hybrid Model	Dwell - Model and Dwell - Test	458979	0.5988
	Vac - Model & Vac - Test	27107	0.0749

Discussion

Improved properties (Capital Values)

The COV and COD show that while the actual assessed values are below market level they are consistent and reliable. The PRD shows the actual assessments and the assessments from all three models show no significant problems with progressivity or regressivity. The actual assessments are superior to the results from three models particularly in term of variability. Of the models the hybrid is superior to both the linear or log-linear model. The actual assessments are carried out by the Valuer General using a computer assisted valuation method (CAV); involve staff in sales analysis to more accurately determine which sales are at market value; may include physical inspection and will normally involve a series of models based on submarkets, and therefore should produce the best results. The hybrid model produces only slightly inferior results. This is impressive considering that these estimates are based on a single model and where sales circumstances and the physical data have not been validated and is inexpensive to implement. Typical sales analysis and data validation would significantly improve these results. Any of the three regression models (linear, log-linear or hybrid) would produce suitable capital value assessments of detached dwellings in the study area.

Vacant Properties (Site Values)

The results for the vacant land models are more variable across the assessment groups. It is the estimate of site values concurrent with capital values and the ability to use improved sales to help estimate the site values that makes the development of hybrid models most useful. The hybrid model produces clearly superior results for the site value assessments. Neither the linear nor log-linear models produce reliable site values. The hybrid model not only outperforms the other two models but also produces superior results to the actual assessed values in terms of COV, COD and PRD. The hybrid model while of complex form uses only a small number of variables where the site component is based on the land area and the LVRS. This suggests that a suitable site value model can be produced from basic data especially when an LVRS can be used. Such a model while ideal for authorities that use both capital and site value could also be useful for rating authorities that only use site or unimproved value and particularly in thin markets. Where transactions of land occur infrequently and are considered to be “questionable” because of this lack of activity, authorities could combine data from rating rolls with data from other sources (such as real estate advertisements) to build hybrid models that use both improved and improved sales. This should produce statistically superior results to the use of statistically untested methods such as the summation approach (where the depreciated cost of replacement is deducted from transaction prices to find a “land value”). This question is the subject of ongoing research.

Improvements to the models

As mentioned earlier the validation of the circumstances of sale and of the physical characteristics of the properties involved would lead to significant increases in predictive accuracy. In each category a small number of observations with very large errors leads to lower than expected overall assessment reliability. Adoption of the ex post procedures used by both O’Conner and Gloudemans would undoubtable improve the tests statistics but may not be reflected in actual final assessments.

One clear problem of the models is the omission of some key variables. In particular the addition of a site features variable would contribute to the model. The data set contained only one useful indicator of site value which was the land area. While this is undoubtedly important this suggests that all vacant properties of the same size and in the same general location will sell for the same amount. Anecdotally we know that issues such as access, corner allotments and main roads will also significantly affect values. For improved properties other site features such as gardens, shedding and features such as swimming pools will also affect value. The inclusion of other variables is supported by Rossini (1998) who found that in Morphett Vale and Woodcroft, regression models were improved by adding additional data such as a site features rating to that held on the standard sales history file.

Conclusions

This study aimed to investigate the usefulness of hybrid AVM's to estimate both capital and site values and establish if such a model could outperform more conventional methods of assessment. The results suggest that these models would be a very useful addition to the armoury of techniques available for mass appraisal. While the predictive accuracy of the hybrid model is only slightly better than the more simple models for capital values, it was far superior in terms of site values and in this regard was also better then the actual assessed values. These models also have the added advantage of being specified in a manner that is more theoretically acceptable to some analysts and is certainly better for estimating value "components" which are typically used when updating valuations after minor renovations. The model can be used to estimate both capital and site values with good accuracy and is particularly useful for estimating site values in situations where there is a scarcity of vacant land transactions.

The use of hybrid models to incorporate a wider range of market transactions when assessing site value makes these models particularly useful in situations where legislation or legal precedent requires valuers to consider the use of improved properties when assessing site values. The ability of the models to estimate the land value effects across both improved and unimproved properties means that these models should be the focus of further research in jurisdictions where site or unimproved values are used.

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Appendix

Figure 1 - Metropolitan Adelaide showing Study Area

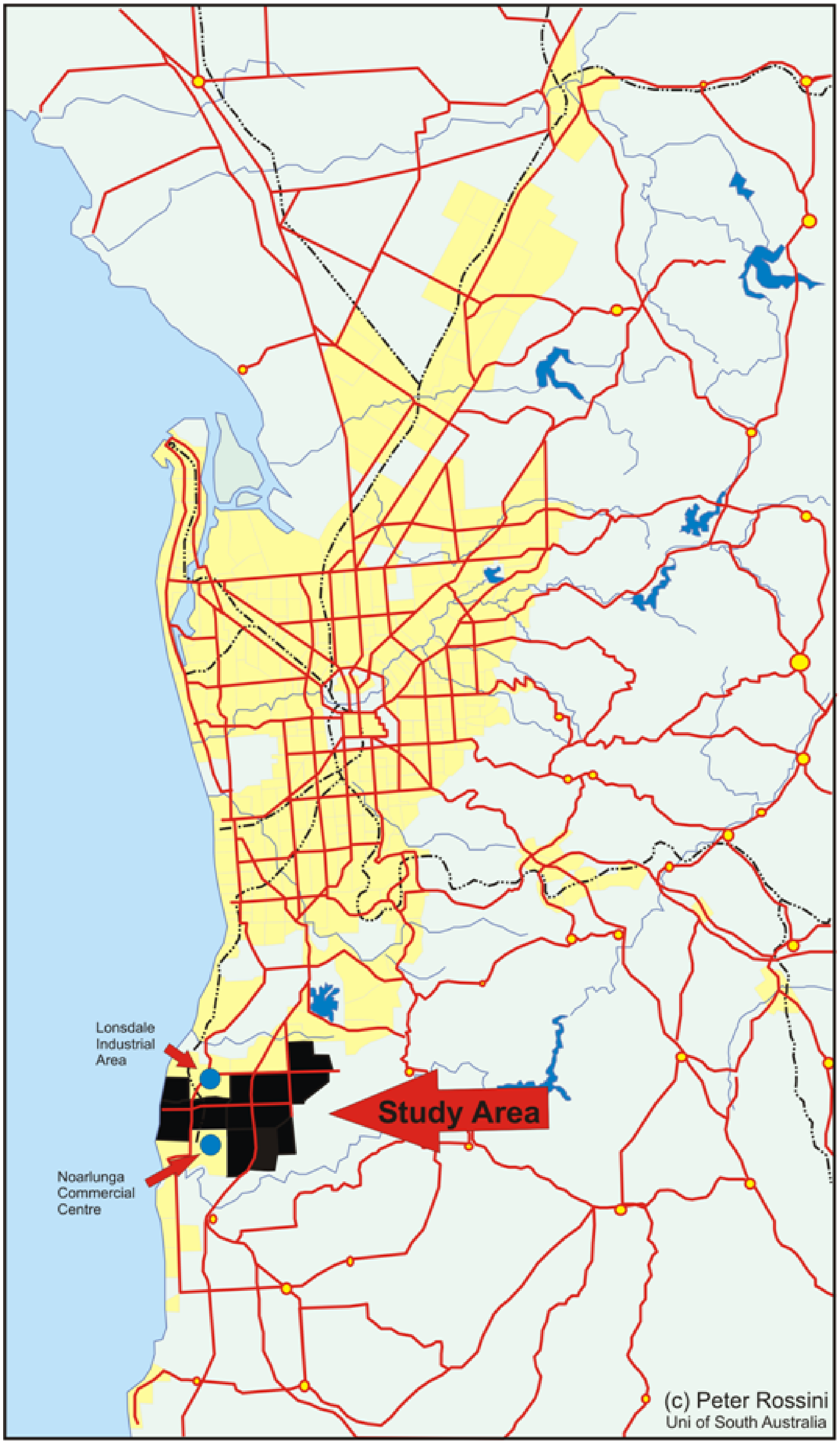


Table 9 - Summary of data by Suburb and Land Use

Suburb	Land Use			Total
	Dwelling - established	Dwelling - not established	Vacant	
CHRISTIE DOWNS	141	1	1	143
CHRISTIES BEACH	172	1	12	185
HACKHAM	123	2	7	132
HACKHAM WEST	102	1	1	104
HUNTFIELD HEIGHTS	125	6	6	137
MORPHETT VALE	789	14	57	860
ONKAPARINGA HILLS	61	13	27	101
O'SULLIVAN BEACH	64	0	3	67
WOODCROFT	301	82	255	638
Total	1878	120	369	2367

Table 10– Initial Regression using all 1998 sales - residuals input to LVRS model

Model Summary(b)

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.927(a)	.860	.857	11644.199

ANOVA(b)

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	883144885392.545	22	40142949336.025	296.067	.000(a)
	Residual	144264964400.164	1064	135587372.557		
	Total	1027409849792.709	1086			

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	22882.485	1598.201		14.318	.000		
	LArea	25.456	2.369	.134	10.747	.000	.852	1.174
	Bage	-625.084	35.347	-.248	-17.684	.000	.671	1.490
	BArea	471.669	6.855	.891	68.802	.000	.787	1.271
	tfwall	19858.178	11854.788	.020	1.675	.094	.966	1.036
	stwall	4813.468	2585.014	.023	1.862	.063	.865	1.157
	AFrame	7156.649	11663.593	.007	.614	.540	.998	1.002
	Archit	8062.828	5257.391	.018	1.534	.125	.986	1.015
	Auster	3885.536	4843.232	.010	.802	.423	.831	1.203
	Bungalo	2025.291	4806.312	.005	.421	.674	.984	1.017
	Colonial	3562.336	1743.133	.024	2.044	.041	.954	1.049
	Contemp	-4201.125	2559.168	-.021	-1.642	.101	.816	1.226
	SAHT	-9266.619	1044.924	-.110	-8.868	.000	.854	1.171
	Cottage	1833.674	6894.817	.003	.266	.790	.953	1.049
	Homested	-6012.764	5382.842	-.013	-1.117	.264	.940	1.064
	Medteran	566.491	5237.213	.001	.108	.914	.993	1.007
	Ranch	-6099.913	2472.148	-.029	-2.467	.014	.985	1.015
	Shack	7758.033	6717.505	.015	1.155	.248	.754	1.326
	Spanish	-5751.673	3555.576	-.019	-1.618	.106	.985	1.015
	Tudor	28496.633	8257.459	.040	3.451	.001	.996	1.004
	Villa	8209.367	2911.635	.038	2.820	.005	.742	1.348
	GIRoof	5394.924	1891.702	.042	2.852	.004	.594	1.682
	ImTilRof	2493.428	5870.279	.005	.425	.671	.791	1.265

a Dependent Variable: SalePrice

Table 11 – Trend Surface Regression (LVRS)

Model Summary(b)

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.441(a)	.194	.187	.89221112

ANOVA(b)

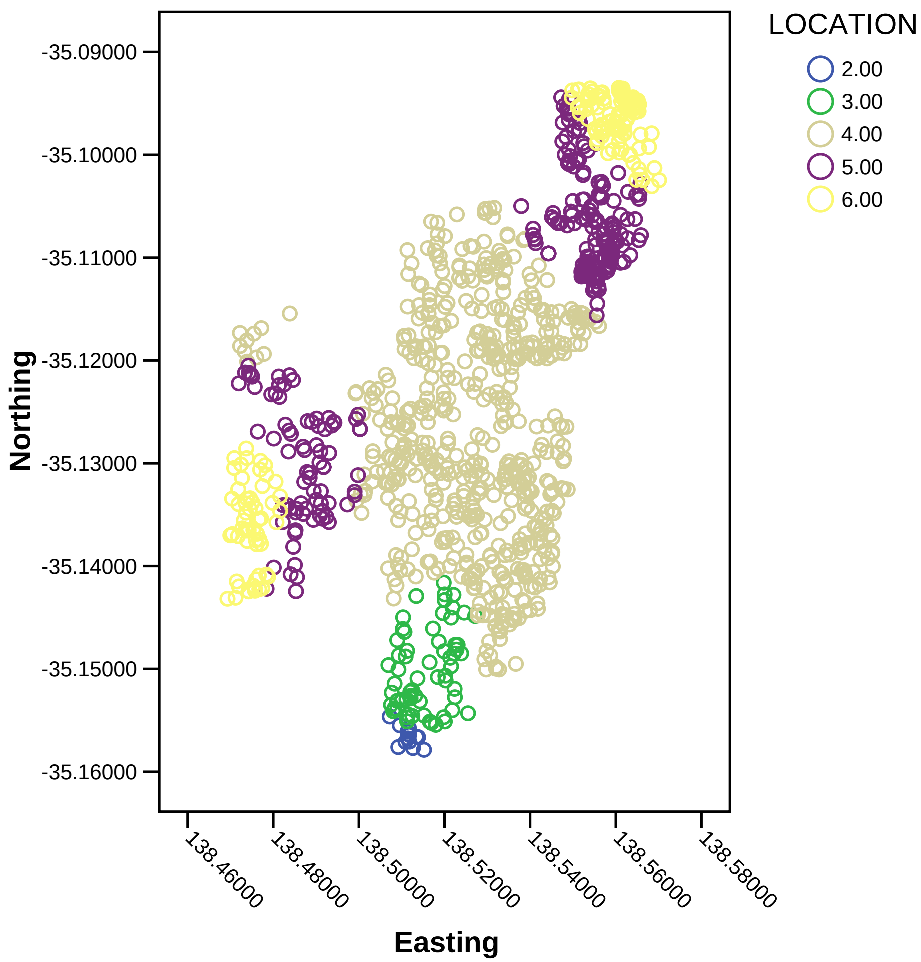
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	206.664	9	22.963	28.846	.000(a)
	Residual	857.336	1077	.796		
	Total	1064.000	1086			

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-6.455	1.929		-3.346	.001
	XCord	1.513	.471	3.866	3.215	.001
	YCord	3.632	1.175	6.074	3.091	.002
	XSqd	-.099	.061	-2.798	-1.633	.103
	YSqd	-.492	.225	-4.947	-2.189	.029
	XbyY	-.710	.168	-7.058	-4.228	.000
	YSqdX	.059	.017	3.249	3.532	.000
	XSqdY	.028	.007	1.971	3.826	.000
	XCubed	.002	.003	.640	.729	.466
	YCubed	.013	.013	.728	.969	.333

a Dependent Variable: Standardized Residual

Figure 2 – Land Value Response Surface



Note: LOCATION is a recoded standardised residual value where 4 = mean, 2 = lowest values 2 standard deviations below the mean and 6 = 2 standard deviations above the mean.

Table 12 – Linear Regression Model

Model Summary(m)

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
12	.932(l)	.869	.867	11201.073

ANOVA(m)

Model		Sum of Squares	df	Mean Square	F	Sig.
12	Regression	892661463211.484	12	74388455267.624	592.907	.000(l)
	Residual	134748386581.224	1074	125464047.096		
	Total	1027409849792.709	1086			

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
12	(Constant)	6522.479	2088.481		3.123	.002		
	Barea_sqrd	.575	.105	.226	5.485	.000	.072	13.875
	Averoomsize	327.510	144.332	.102	2.269	.023	.060	16.617
	LOCATION	14913.598	853.122	.212	17.481	.000	.834	1.199
	SAHT	-12170.556	984.951	-.145	-12.357	.000	.889	1.125
	BArea	303.467	38.997	.573	7.782	.000	.022	44.456
	LArea	16.612	2.239	.087	7.421	.000	.882	1.134
	Villa	10407.890	2728.527	.048	3.814	.000	.782	1.279
	Contemp	-14549.768	2397.212	-.072	-6.069	.000	.860	1.162
	Tudor	26823.017	7963.010	.037	3.368	.001	.991	1.009
	Shack	-19541.314	5715.639	-.038	-3.419	.001	.964	1.038
	Auster	-16017.973	4515.820	-.042	-3.547	.000	.885	1.130
	GIRoof	5344.418	1746.402	.042	3.060	.002	.645	1.550

a Dependent Variable: SalePrice

Table 13 - Log-linear Regression Model

Model Summary(m)

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
12	.938(l)	.880	.879	.14267

ANOVA(m)

Model		Sum of Squares	df	Mean Square	F	Sig.
12	Regression	160.937	12	13.411	658.883	.000(l)
	Residual	21.861	1074	.020		
	Total	182.799	1086			

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
12	(Constant)	10.077	.027		378.831	.000		
	BArea	.008	.000	1.118	15.890	.000	.022	44.456
	LOCATION	.202	.011	.215	18.574	.000	.834	1.199
	Barea_sqrd	-1.105E-05	.000	-.326	-8.284	.000	.072	13.875
	SAHT	-.153	.013	-.136	-12.164	.000	.889	1.125
	LArea	.000	.000	.103	9.133	.000	.882	1.134
	Contemp	-.155	.031	-.058	-5.090	.000	.860	1.162
	Villa	.090	.035	.031	2.585	.010	.782	1.279
	Auster	-.215	.058	-.042	-3.744	.000	.885	1.130
	Shack	-.228	.073	-.034	-3.138	.002	.964	1.038
	Averoomsize	.004	.002	.091	2.110	.035	.060	16.617
	GIRoof	.047	.022	.028	2.098	.036	.645	1.550
	Tudor	.202	.101	.021	1.994	.046	.991	1.009

a Dependent Variable: lnSalePrice

Table 14– Hybrid (Non-linear regression) Model

Nonlinear Regression Summary Statistics   Dependent Variable SalePrice

R squared = 1 - Residual SS / Corrected SS =   .88558

ANOVA

Source	DF	Sum of Squares	Mean Square
Regression	15	7.87191E+12	5.24794E+11
Residual	1072	1.1578E+11	108003773.5
Uncorrected Total	1087	7.98769E+12	
(Corrected Total)	1086	1.01185E+12	

Nonlinear Regression Equation

Price = (b0*LOCATION ** b1*LAREA ** b2)+((b4*BArea **
b5*BAge**b6*Condition**b16)*(b9**SAHT*b10**AUSTER*b11
Contemp*b12shack*b13**bungalo*b14**tudor*b15**villa)) .

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval Lower	Upper
b0	1966.359656	0.027508272	1966.30568	1966.4136
b1	0.358951813	0.010707846	0.337941098	0.3799625
b2	0.448561023	0.033399176	0.383025849	0.5140962
b4	132.3595789	0.058728584	132.2443429	132.47481
b5	1.038000533	0.037376536	0.964661065	1.11134
b6	-0.147135939	484.4035473	-950.6337892	950.33952
b7	0.9	0.10956341	0.685016935	1.1149831
b9	0.743331059	0.16601849	0.417573001	1.0690891
b10	0.993005357	0	0.993005357	0.9930054
b11	0.833784552	0.144037227	0.551157676	1.1164114
b12	0.768148267	43.41790016	-84.42556025	85.961857
b13	0.954399332	0.120377797	0.7181965	1.1906022
b14	1.339034897	0.021487183	1.296873189	1.3811966
b15	1.140268045	0.030264685	1.080883305	1.1996528
b16	0.648493763	0.046055122	0.558125353	0.7388622

Table 15 - Ratio Study Statistical Analysis - Linear and Log-linear Models

	Linear Model				Log-Linear Model			
Sample Statistics	Dwell - Model	Dwell - Test	Vac - Model	Vac - Test	Dwell - Model	Dwell - Test	Vac - Model	Vac - Test
Number of observations used	891	1,061	182	181	895	1,061	183	179
Total appraised value	$79,222,313	$94,846,233	$7,043,179	$6,726,133	$79,355,138	$94,846,233	$6,919,175	$6,618,298
Total market value	$79,480,063	$100,456,158	$7,028,000	$7,165,170	$79,732,563	$100,456,158	$7,123,500	$7,126,170
Average appraised value	$88,914	$89,393	$38,699	$37,161	$88,665	$89,393	$37,810	$36,974
Average market value	$89,203	$94,681	$38,615	$39,587	$89,087	$94,681	$38,926	$39,811
Sales Ratio Tracking								
Slope	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R squared	0.005	0.065	0.000	0.003	0.003	0.065	0.010	0.001
Assesment Ratios								
Mean Ratio	1.010	0.959	1.039	0.989	1.007	0.959	1.024	0.989
Geometric Mean Ratio	1.004	0.952	1.021	0.967	1.001	0.952	1.000	0.963
Weighted Mean Ratio	0.997	0.944	1.002	0.939	0.995	0.944	0.971	0.929
Median Ratio	0.998	0.949	1.020	0.984	0.998	0.949	0.999	0.974
First Quartile	0.929	0.879	0.892	0.817	0.927	0.879	0.858	0.801
Third Quartile	1.086	1.036	1.192	1.140	1.083	1.036	1.167	1.161
Interquartile Range	0.157	0.157	0.300	0.323	0.156	0.157	0.310	0.360
Upper trim point	1.322	1.272	1.641	1.625	1.318	1.272	1.632	1.700
Lower trim point	0.694	0.643	0.442	0.332	0.693	0.643	0.394	0.262
Price-related differential (PRD)	1.013	1.015	1.037	1.054	1.012	1.015	1.054	1.065
Coefficient of Dispersion (COD)	9.3%	9.6%	15.8%	17.8%	9.1%	9.6%	17.9%	19.5%
Standard deviation	0.115	0.113	0.197	0.209	0.114	0.113	0.221	0.227
Coefficient of variation (COV)	11.4%	11.8%	19.0%	21.1%	11.3%	11.8%	21.6%	23.0%
Minimum Ratio	0.695	0.648	0.608	0.447	0.698	0.648	0.436	0.400
Maximum Ratio	1.320	1.268	1.529	1.601	1.313	1.268	1.594	1.620
Range of Ratios	0.625	0.621	0.921	1.153	0.614	0.621	1.159	1.220
Probability that population mean ratio is between 90% and 110%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
95% mean two-tailed confidence interval	1.001 to 1.019	0.951 to 0.966	1.007 to 1.072	0.954 to 1.024	0.999 to 1.016	0.951 to 0.966	0.987 to 1.06	0.951 to 1.027
95% median two-tailed confidence interval	0.989 to 1.006	0.941 to 0.957	0.987 to 1.052	0.95 to 1.019	0.989 to 1.006	0.941 to 0.957	0.962 to 1.035	0.936 to 1.012
95% weighted mean two-tailed confidence interval	0.988 to 1.005	0.936 to 0.952	0.969 to 1.035	0.904 to 0.974	0.987 to 1.004	0.936 to 0.952	0.935 to 1.008	0.891 to 0.967
Kurtosis	-0.191	-0.233	-0.588	-0.432	-0.144	-0.233	-0.272	-0.602
Skew	0.303	0.200	0.247	0.277	0.312	0.200	0.413	0.258
Number trimmed (being outliers)	26	20	4	2	22	20	3	4

Table 16 - Ratio Study Statistical Analysis - Hybrid Model and Actual Assessed Values

	Hybrid Model				Actual Assessed			
Sample Statistics	Dwell - Model	Dwell - Test	Vac - Model	Vac - Test	Dwell - Model	Dwell - Test	Vac - Model	Vac - Test
Number of observations used	879	1,059	173	168	900	1,049	179	161
Total appraised value	$76,793,828	$92,502,897	$6,835,783	$6,409,809	$74,236,571	$86,568,281	$6,295,000	$5,461,000
Total market value	$78,496,413	$100,034,100	$6,807,050	$6,770,970	$80,258,563	$99,404,550	$6,905,500	$6,601,220
Average appraised value	$87,365	$87,349	$39,513	$38,154	$82,485	$82,525	$35,168	$33,919
Average market value	$89,302	$94,461	$39,347	$40,303	$89,176	$94,761	$38,578	$41,001
Sales Ratio Tracking								
Slope	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R squared	0.003	0.061	0.015	0.020	0.012	0.078	0.063	0.006
Assessment Ratios								
Mean Ratio	0.988	0.937	1.024	0.972	0.935	0.884	0.918	0.855
Geometric Mean Ratio	0.984	0.931	1.017	0.963	0.931	0.879	0.915	0.833
Weighted Mean Ratio	0.978	0.925	1.004	0.947	0.925	0.871	0.912	0.827
Median Ratio	0.985	0.931	1.000	0.962	0.925	0.880	0.933	0.838
First Quartile	0.920	0.863	0.931	0.877	0.878	0.827	0.857	0.714
Third Quartile	1.052	1.008	1.108	1.063	0.991	0.941	0.970	0.966
Interquartile Range	0.132	0.145	0.177	0.186	0.113	0.115	0.113	0.251
Upper trim point	1.249	1.226	1.374	1.341	1.160	1.113	1.138	1.342
Lower trim point	0.722	0.645	0.665	0.598	0.709	0.655	0.688	0.337
Price-related differential (PRD)	1.010	1.014	1.020	1.027	1.010	1.015	1.007	1.033
Coefficient of Dispersion (COD)	8.0%	9.3%	10.0%	11.4%	7.3%	8.1%	6.8%	18.5%
Standard deviation	0.098	0.108	0.126	0.137	0.085	0.090	0.079	0.194
Coefficient of variation (COV)	9.9%	11.5%	12.3%	14.0%	9.0%	10.1%	8.6%	22.7%
Minimum Ratio	0.725	0.650	0.768	0.678	0.712	0.656	0.691	0.469
Maximum Ratio	1.249	1.225	1.349	1.319	1.159	1.113	1.099	1.310
Range of Ratios	0.524	0.575	0.581	0.641	0.448	0.457	0.408	0.840
Probability that population mean ratio is between 90% and 110%	100.0%	100.0%	100.0%	100.0%	100.0%	0.0%	99.9%	0.2%
95% mean two-tailed confidence interval	0.981 to 0.996	0.93 to 0.945	1.003 to 1.046	0.949 to 0.996	0.928 to 0.941	0.877 to 0.89	0.905 to 0.931	0.821 to 0.889
95% median two-tailed confidence interval	0.978 to 0.992	0.924 to 0.939	0.979 to 1.022	0.938 to 0.985	0.919 to 0.931	0.874 to 0.886	0.92 to 0.947	0.804 to 0.872
95% weighted mean two-tailed confidence interval	0.971 to 0.986	0.917 to 0.932	0.983 to 1.026	0.923 to 0.97	0.919 to 0.931	0.865 to 0.877	0.898 to 0.925	0.793 to 0.862
Kurtosis	-0.255	-0.258	-0.332	-0.400	-0.231	-0.337	-0.148	-0.374
Skew	0.176	0.153	0.553	0.326	0.231	0.050	-0.309	0.371
Number trimmed (being outliers)	37	21	13	15	16	31	7	22