A PARAMETRIC MODEL OF THE EX ANTE DIRECT REAL ESTATE RISK AND RETURN ESTIMATION

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Abstract

This paper investigates the merits of a unique real estate risk-and-return estimation model. In this model, we rigorously integrate the bond duration-convexity concepts, the real estate return Beta distribution function and the real estate equivalent yield valuation model. Only very limited information through the lease structure of a direct real estate asset is required for this model of risk and return estimation. No historical data is utilized in estimating the real estate risk and the expected return via this model. Empirically this model offers a useful and innovative approach to the risk-and-return estimation of new direct real estate assets, which do not have past time series.

Key Words: Ex Ante, Direct Real Estate, Risk and Return.

Introduction

Risk and return are inherent elements in the investment of a direct real estate asset, and the investment prerequisite is that an initial capital outlay requires an expected (*i.e.* a minimum required) flow of future income. The future income flow may well be indefinite and there is an inherent risk in the investment. Risk refers to the variability of returns that is associated with an investment owing to movements in financial variables, and is usually measured by the standard deviation (σ) or volatility of unexpected outcomes. There is also an inherent risk in real estate markets owing to the mismatch in timing between new supply and demand shocks. This paper focuses more on how the level of risk will change as the level of variability in the financial variable movement changes.

Thus there is a need for investors to ascertain the level of risk in direct real estate assets. The measurement of real estate risk, in conformity with modern portfolio theory (MPT), should reflect an investor's *ex ante* expectations, rather than to focus on what has happened in the past. One of the most important assumptions of the Capital Asset Pricing Model (CAPM) theory is that investors base their decisions on expected return and risk and the basic tenet of the MPT theory is to maximize the expected return and minimize the standard deviation of the return. Historic measures of risk are merely helpful in forecasting expected risk under set scenarios. Owing to limited empirical data availability, statistical techniques such as sensitivity analysis, probability and also Monte Carlo simulation are typically used in real estate risk assessment to overcome this problem. Real estate structural risk factors can even be further investigated via these methods, and this is of much significance since the real estate risk factors in turn affect the portfolio return. Therefore this paper views risk purely from the aspect of investors' expectations upon which lays the foundation of MPT. The historical measures of risk are only useful under very contrived or controlled scenarios, where the alternatives are clear and experiments can conceivably be repeated. However, very often the "past" may well not be an accurate predictor of the future and corresponding expected returns. Although most of the current principal investment asset classes or markets have well established time series of returns that can be used in the estimation of future expected risks and returns, this, however, is not case for new products in the real estate markets. Thus, an estimation of the expected risk and return is indispensable in direct real estate investment.

This research main objective is to rigorously integrate the bond duration-convexity concept, the beta distribution function and the real estate equivalent yield valuation model for the purpose of better risk estimation and definition, where limited information for a specific direct real estate asset is available. The integrated risk model is able to estimate several key direct real estate risk measures, via a beta distribution sub-model that includes kurtosis, the mean absolute deviation, the Sharpe ratio, value at risk, low partial moment of the n^{th} order, and the direct real estate asset duration and convexity. These risk-measure estimations are in turn utilized to derive the expected total returns (and the expected capital values) at the level of the direct real estate asset. The resulting low partial moment risks, the direct real estate asset duration and convexity provide further insights into the nature of the risk-adjusted return for a specific and direct real estate asset but on the basis of limited and available information for rents, real estate sector yields, the change in yields, lease maturity period and the risk-free rate of interest. In this way, the direct real estate structural risk factors can be further investigated, and this is of much significance since such risk factors in turn affect the portfolio return, say, the internal rate of return (IRR). The subsequent investigation of a certain key direct real estate risk factor, while controlling the other remaining risk factors, will surely shed light on the risk behavior of the real estate portfolio return, and should offer better insights to enable the investor to adjust his exposure to the volatility of a direct real estate risk factor (*i. e.* the risk source).

The integrated direct real estate risk-measure estimation model utilizes the Jones Lang LaSalle Real Estate Information Service-Asia (JLL REIS-Asia), for the prime office sector in Singapore and Hong Kong for year 2002. As a result, this paper hypothesizes that the expected total returns and expected capital values for a direct real estate asset are principally generated from the following parameters:

- Passing rent
- Current rental value
- Expected yields and yield changes
- Lease maturity period
- Risk-free rate of interest

The rest of this research is divided into several sections. After the introduction comes the review of the related literature pertaining to modeling the estimates of the direct real estate expected risks and expected returns. Section two is the theoretical framework of analysis, which covers the duration model, the real estate asset volatility relative to a market index, the real estate duration and its measurement, as well as the low partial moment (LPM) risk. Section three investigates the development of the integrated direct real estate risk estimation model and the corresponding return model on an *ex ante* basis but within the comparative context of two real estate markets – the prime Singapore office market and the prime Hong Kong office market. Section four is concerned with a comparative examination of the model results and those obtained from a structured Monte Carlo simulation model. The final section of this research is the conclusion with recommendations for future research.

The Theoretical Framework of Analysis

The Modified Duration Model

Duration (D_t) is frequently used in the bond market to match asset liabilities. It measures the sensitivity of the value of an asset to changes in the interest rate. It is firstly developed by Macaulay (1938) and formulated as follows:

$$\frac{dV_t}{dy_t} \times \frac{1}{V_t} = \frac{-D_t}{(1+y_t)} \tag{1}$$

Where,

V_t : the value of the asset at time t;

dy_t : the change in discount rate at time t.

The expression on the right hand side of equation (1), $\frac{-D_t}{(1+y_t)}$, is referred to as the modified duration D_t*. Rearranging the formula, the asset value's growth rate is obtained in terms of the modified duration as follows[1]:

$$\frac{dV_t}{V_t} = -D_t * dy_t \tag{2}$$

The Real Estate Return Volatility Relative to a Market Index

The anticipated rate of return of a real estate asset (property) j over a short period can be initially expressed as[2]:

$$R_{jt} = \frac{V_{jt} + a_{jt} + dV_{jt}}{V_{jt}} \tag{3}$$

, where a_{jt}: the initial income at time t;

V_{jt}: the real estate asset value at time t;

dV_{jt}: the anticipated change in value at time t.

[1] The link between the bond price volatility and duration is firstly developed by Fisher (1966) and Hopewell and Kaufman (1973) later extended its discrete form.

[2] This anticipated rate of return is estimated similarly as the anticipated rate of return on a default-free bond over a short interval, for further details, see Livingston (1978).

Substituting from equation (2) for the real estate asset j gives

$$R_{jt} = 1 + \frac{a_{jt}}{V_{jt}} - D_{jt} * dy_{jt} \qquad (4)$$

Based on the data for the later empirical analysis, the variance of the term a_{jt}/V_{jt} a is very small, for the capital value of the real estate asset, V_{jt} , is much larger comparing with its initial income a_{jt},. As we can see later in the data, the variance of a_{jt}/V_{jt} for the Singapore prime office, prime retail, luxury residential sectors (which consist of our research data) and the weighted real estate market are respectively 0.0083%, 0.0090%, 0.0090% and 0.0088%. Thus, the variance of the term, a_{jt}/V_{jt}, can be neglected in the calculation of the variance of R_{jt}. Hence, the variance of the stochastic variable R_{jt} equals to the variance of the product of two stochastic variables D_{jt}* and dy_{jt}, which are correlated. When one or both of the coefficients of variation of the two stochastic variables are relatively small, the usual approximate formula for the variance of the product of the two stochastic variables X and Y, Var(X*Y) is as follows[3]:

$$Var(X * Y) = Var(X) * [Exp(Y)]^2 + Var(Y) * [Exp(X)]^2 \qquad (4a)$$

Since the coefficient of the variation of dy_{jt} is relatively small for all the three real estate sectors (i.e. the Singapore prime office, prime retail and luxury residential sector, details please see the data) and weighted real estate market, we can take this approximate formula to measure the variance of R_{jt}. Therefore:

$$Var(R_{jt}) = Var(D_{jt}) * [Exp(dy_{jt})]^2 + Var(dy_{jt}) * [Exp(D_{jt}^{*})]^2 \qquad (4b)$$

With a further investigation of the data, the author find the mean square of dy_{jt} for the Singapore prime office, prime retail, luxury residential and the weighted real estate market are almost trivial (respectively 0.007066%, 0.0025%, 0.000038% and 0.000438%) comparing with their respective duration mean square (which are around 100 ~ 900 for the duration mean is around 10~30 years). In the calculation of the

[3] On the exact variance of stochastic variables, please see Goodman (1960).

variance of R_{jt} , we can neglect the first part of the right hand side of *eq* (4b). So we further get $Var(R_{jt}) = Var(dy_{jt}) * [Exp(D_{jt}^{*})]^2$. When calculating the variances of R_{jt} and dy_{jt} and the expectation of D_{jt}^{*} , we take the moving average and the measurement of duration is in an expectational form. For simplicity in notation, we get that at time *t,* the variance of R_{jt} in Equation (4):

$$Var(R_{jt}) = D_{jt}^{*2} Var(dy_{jt}) \quad (5)$$

A similar expression also exists for the variance of an index of real estate market movements R_{mt} such that

$$Var(R_{mt}) = (D_{mt}^{*})^2 Var(dy_{mt}) \quad (6)$$

The single index model suggests that the volatility of an investment relative to an index can be expressed as follows:

$$\beta_{jt} = \frac{\text{cov}(R_{jt}, R_{mt})}{Var(R_{mt})} \quad (7)$$

This can be written as

$$\beta_{jt} = \frac{\rho(R_{jt}, R_{mt})\sigma(R_{jt})\sigma(R_{mt})}{Var(R_{mt})} \quad (8)$$

By substitution, the following expression is obtained:

$$\beta_{jt} = \frac{D_{mt} * D_{jt} * \rho(R_{jt}, R_{mt})\sigma(dy_{jt})\sigma(dy_{mt})}{(D_{mt}*)^2 Var(dy_{mt})} \quad (9)$$

Simplifying gives

$$\beta_{jt} = \frac{D_{jt}*}{D_{mt}*} \frac{\rho(dy_{jt}, dy_{mt})\sigma(dy_{jt})}{\sigma(dy_{mt})} \quad (10)$$

Eq (10) shows that the duration can play a theoretical role in determining the risk of a direct real estate asset investment and provides a rationale for non-stationarity of betas. According to *eq* (10), the volatility of a direct real estate asset relative to a real estate market index is made up of two components. The first component is the modified duration of the property (*i. e.* the real estate asset) divided by a similar duration term for the real estate index (market duration). The second component is the covariance of changes in the equivalent yield of the direct real estate asset relative to the changes in the real estate market yield. This latter expression can also be interpreted to be the volatility of changes in the real estate yield. So, Equation (10) can be re-expressed as:

$$\beta_{jt} = \frac{D_{jt}*}{D_{mt}*}\beta_{dy_{jt}.dy_{mt}} \tag{11}$$

Note that equation (11) provides an estimate of β_{jt} that is measured relative to a real estate market index. The justification for this approach is that real estate investors are frequently concerned about how well their portfolios perform relative to the real estate market. Via *eq* (11), we can estimate the volatility of the real estate asset (or sector) return relative to the market that is useful in the performance measurement of the direct real estate portfolio. If the real estate index represents a reasonable proxy for the whole real estate market, and assuming equilibrium conditions, then there would be a linear relationship between the expected risk premium for both the real estate market and the market portfolio. This would imply that equation (11) can be used to estimate the real estate systematic risk within a capital market framework.

The advantage of equation (11) in estimating the volatility of the real estate (or sector) return, relative to the market, is that it does not rely on a time series of historical data, and can be expressed in expectation form. As the duration is estimated from available data, the volatility of a real estate asset (or sector) can be readily estimated whenever a valuation is undertaken.

Estimation of the volatility of the real estate (or sector) return, relative to the market, via equation (11), offers us some meaningful insights. Equation (11) reveals that the β of a direct real estate asset's return depends on the relative size of the duration of the direct real estate asset and the real estate market as well as the volatility of changes in the real estate yields. The importance of the latter implication is well observed in the valuation of an over rented real estate asset within the context of the United Kingdom (UK) practice. In this instance, a valuer may well argue that over an agreed time horizon, there would be changes in the market yield appropriate to the real estate asset so that the covariance between yield changes would be close to zero. As a result, β_j is also close to zero even though the respective durations take on positive values. The inference of this result is that in a capital market framework, the appropriate discount rate at which to value the real estate asset should be close to the risk free rate of return. In practice, we see over-rented properties being valued using the return on long-term government bond in 1990s in UK.

The Direct Real Estate Duration & Its Measurement

To use equation (11), the estimation of the duration of a direct real estate asset is prerequisite. Based on equation (2), the modified duration of the direct real estate asset *j* at time *t* can be formulated as:

$$-D_{jt}^* = \frac{dV_{jt}}{dy_{jt}} \times \frac{1}{V_{jt}} \tag{12}$$

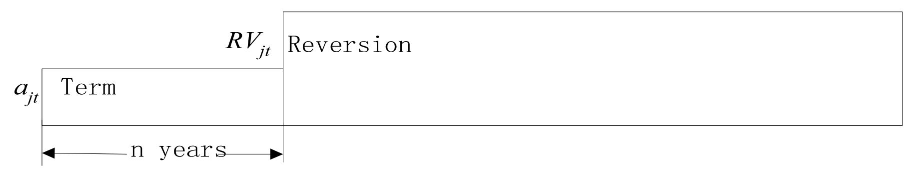
The direct real estate asset value, V_{jt}, can be estimated from the present value of the typical term and reversion freehold valuation model. The 'typical term' is represented by an initial income stream, a_{jt}, that is fixed for n years at which time it is reviewed to the open market yield value, RV_{jt}. The present value is found by discounting at the equivalent yield, y_{jt}. Fig 2 depicts the equivalent yield model for a direct real estate asset in two parts. The first part consists of the current annual rental income a_{jt} for n years until the next rent review. The second part occurs at the next rental review when the annual rental income is replaced by the current estimate of rental value, RV_{jt}, which is then assumed to remain constant in perpetuity. For the direct real estate asset *j*, the present value at time *t*, V_{jt}, can be expressed as:

$$V_{jt} = a_j\left[\frac{1-(1+y_{jt})^{-n}}{y_{jt}}\right] + \frac{RV_{jt}}{y_{jt}(1+y_{jt})^n} \qquad (13)$$

Rearranging gives

$$V_{jt} = \frac{a_{jt}}{y_{jt}} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n} \qquad (14)$$

Fig 1.2 The Term and Reversion Parts of the Direct Real Estate Asset Equivalent Yield Model



(Source:Author, 2005)

Equation (14) takes a non-linear form and is known as the real estate equivalent yield model, which is the most common method used for valuing the commercial real estate asset (*i.e.* property) and for analyzing current transactions. The equivalent yield in *eq* (14) is usually lower than the risk adjusted return, reflecting the fact that there is growth in the income stream. In this model the equivalent yield as a discount rate for the expected cash flow incorporates the specific risk characteristics of the real estate asset, such as the lease term, rental growth, the physical condition and even the investor's expectation of the economy such as inflation expectation, forecasts of economy, and expected depreciation.

While using the real estate equivalent yield model, it is the UK practice and throughout many of the British Commonwealth countries, including Singapore, to set RV_{jt} equal to the current rental value even though it arises *n* periods in the future. The equivalent yield incorporates readily available information that is expressed in current day terms. In a market that is yield driven[4], it may well be reasonable to assume that most valuers are familiar with equivalent yields, and the equivalent yields embody adequate information with respect to the lease structure of individual real estate assets, together with the expectations of rental value growth and expected returns.

It should be firstly noticed that although equation (14) can be shown to be misspecified[5] in economic terms, there is no guarantee that it would produce valuations that differ from a model that explicitly allows for growth in rental values. The choice of the yield in these models is vital. Because of the importance of the direct real estate equivalent yield, valuers are interested to know by how much a small change in the yield can affect capital value. It is thus appropriate to examine the duration of a direct real estate asset relative to changes in the equivalent yield.

From equation (14), the first derivative of V_{jt} with respect to y_{jt} can be expressed as:

$$\frac{dV_{jt}}{dy_{jt}} = -\frac{a_{jt}}{y^2_{jt}} - \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n}[\frac{1}{y_{jt}} + \frac{n}{(1+y_{jt})}] \quad (15)$$

Dividing through by the real estate asset value V_{jt} and substituting $1/V_{jt}$ by $\frac{y_{jt}(1+y_{jt})^n}{a_{jt}(1+y_{jt})^n + (RV_{jt} - a_{jt})}$ would give the modified duration as:

$$D^*_{jt} = \left\{ \frac{a_{jt}}{y^2_{jt}} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n} \left[\frac{1}{y_{jt}} + \frac{n}{(1+y_{jt})} \right] \right\} \bullet \frac{y_{jt}(1+y_{jt})^n}{a_{jt}(1+y_{jt})^n + (RV_{jt} - a_{jt})} \quad (16)$$

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[4] In a market that is not just yield driven, where asset value change is significant, the change of the equivalent yields also captures these asset value variances and thus make this measurement still viable.
[5] Misspecification arises when we set RV_{jt} equal to the current rental value for economic inconsistencies. However, economic deficiencies in the model, as well as differences in the lease structure, are accommodated in the choice of equivalent yield. There are widely publicized equivalent yields with property transactions and at the index level, time series of equivalent yields are also readily available and form an important part of published information for real estate, for this reason, the equivalent yield model is the most common approach used to value property.

Noting that for a fully rack-rented[6] real estate asset in which the rental value, RV_{jt}, is equal to the passing income, a_{jt}, the modified duration can be reduced to

$$D^*_{jt} = \frac{1}{y_{jt}} \qquad (17)^7$$

An alternative approach to better understand *eq* (12) and *eq* (17) is to consider a perpetual floating-rent contract that provides for an initial net rent or cash flow of NR_0, and is being continuously adjusted to fully reflect changes in the level of market rents. A similar duration under the Dividend Discount Model can be obtained. Suppose that the level of market rent grows at an average rate of *g% p.a.* Let *k* denote the risk-adjusted discount rate that is appropriate for discounting the expected cash flows. The present value, *P*, of the cash flows for a perpetual floating-rent contract is defined in *eq* (17a).

$$P = \int_0^\infty NR_t e^{-kt} dt = \int_0^\infty NR_0 e^{-(k-g)t} dt = \frac{NR_0}{(k-g)} \qquad (17a)$$

This is the Gordon-Sharpiro model modified for a direct real estate cash flow stream. We can further express the total differential of the price as a function of instantaneous changes in the discount rate, *k*, the expected growth rate, *g,* and the base rent NR_0:

$$\Delta P = -\frac{NR_0}{(k-g)^2}(\Delta k - \Delta g) \qquad (17b)$$

$$\frac{\Delta P}{P} = -\frac{NR_0}{(k-g)^2}(\Delta k - \Delta g) \div \frac{NR_0}{(k-g)} = -D_{DDM}\Delta k + D_{DDM}\Delta g + u \qquad (17c)$$

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[6] Rack-rent is originally a rent which a property would command in a free market. It is the highest amount that can be paid for land from labor's production that will enable him to survive (and reproduce). Even as new skills and techniques are adopted, and innovative technology is put to work, so will rack-rent rise, swallowing the lion's share of the product. For a fully rack-rented property, where the passing rent equals the rental value, valuers would value the income stream until the review date as an annuity, and would capitalize the increase in rent in perpetuity at the equivalent yield.

[7] For mathematic proof of *eq* (17), please refer to Appendix 1

, where

$\frac{\Delta P}{P}$ = The instantaneous rate of price change for a direct real estate asset;

Δk = Change in the discount rate;

Δg = Change in the expected growth rate of net real estate market rents;

u = Unexpected net rent growth; and

$D_{DDM} = \frac{1}{(k-g)}$ = the duration under the dividend discount model.

It is noteworthy that the alternative *eq* (17c) is essentially consistent with *eq* (2) while the expression, $D_{DDM} = \frac{1}{(k-g)}$, is essentially consistent with *eq* (17), with the latter expression incorporating the expected growth rate of the direct real estate market net rents, g, and its change, Δg, however, in the instance of a fully rack-rented and direct real estate asset, the capitalization factor (or year's purchase), $\frac{1}{y_{jt}}$, is equivalent to the modified duration. It is thus implicit that a 1% shift in yields should result in a change in capital value that is approximately equal to the duration. Such an implicit relationship is approximate because the modified duration model for the fully rack-rented and direct real estate asset assumes that as the direct real estate yields change, the change in capital value is linear. In reality, however, this implicit relationship is curvilinear. In the case of a fully rack-rented real estate asset, the capitalization factor (or year's purchase) is equivalent to the modified duration. It is therefore implicit that a 1% shift in yields should result in a change in capital value that is approximately equal to the duration. The relationship is approximate because the duration model assumes that as the real estate yields change, the change in capital value is linear. In reality, however, this relationship is curvilinear. To illustrate for clarity, consider the value of US$1 capitalized in perpetuity at 6.5% p.a. The capital value of US$ 1 in perpetuity is US$15.38 and the duration, resulting from equation (17), is 15.38 years. If the direct real estate yield drops by 1% to 5.5%, then the capital value of US$1 in perpetuity would be US$18.18, *i.e.* an increase of 18.22% over the original capital value that is more than the value of the duration derived from equation (17). However, if the direct real estate yield increases by 1%, then the capital value would drop to US$13.33, with a drop in value of 13.31% that is less than the value of the duration. The average of these two changes at 15.76% is much closer to the percentage of change, implied by the duration of 15.38 years. Although it is possible to compensate for these changes by taking into consideration the convexity of the value-yield curve, this research's interest in volatility is concerned more with the relative change in duration so that accounting for convexity may not make a substantial difference to the overall estimation.

The Direct Real Estate Asset Total Risk Duration Model

In a duration model, a linear relationship is presumed between changes in both the fixed-income asset value and the market-wide interest rate. For large changes in the interest rate, the model does not accurately reflect changes in value, and such changes can be reflected through the convexity concept. However, by writing the change in the capital value of the direct real estate asset *j* as the first two terms of a Taylor expansion, then the following expression can be derived:

$$dV_{jt} = \frac{dV_{jt}}{dy_{jt}} dy_{jt} + \frac{1}{2} \frac{d^2 V_{jt}}{dy^2{}_{jt}} (dy_{jt})^2 \tag{18}$$

Dividing through by V_{jt} and substituting $D_{jt}*$ for the modified duration and C_{jt} for convexity, would produce the expression:

$$\frac{dV_{jt}}{V_{jt}} = -D^*{}_{jt} dy_{jt} + \frac{1}{2} C_{jt} (dy_{jt})^2 \tag{19}$$

, where $C_{jt} = \frac{d^2 V_{jt}}{dy^2{}_{jt}} \frac{1}{V_{jt}}$

Assuming a fully rack rented direct real estate asset (property), the percentage change in capital value is

$$\frac{dV_{jt}}{V_{jt}} = -\frac{1}{y_{jt}} dy_{jt} + \frac{1}{y_{jt}^2} (dy_{jt})^2 \qquad (20)^8$$

Taking convexity into consideration may improve our calculations and knowing the distribution of the direct real estate yield changes, it would be possible to simulate a distribution for the percentage change in value. However, the main concern in looking at convexity is in the effect that it could have on the direct real estate total risk. This is imperative for large changes in the direct real estate yield. However, the average change in the yield for the Singapore real estate market is only -0.21% per quarter. With such a small value, the effect of convexity only influences the third decimal place in the growth calculations. As long as the direct real estate yield changes are relatively small, then it is likely that convexity would not have a great influence on the estimate of direct real estate total risk, and can thus be ignored. To provide an estimate of the total risk, a further assumption is that the direct real estate asset is fully rented so that the current income is equal to the rental value. Given these simplifications for practical purposes, the direct real estate total risk is expressed as:

$$Var(g_{jt}) = (D_{jt}*)^2 Var(dy_{jt}) \qquad (21)$$

, where $Var(g_{jt})$ is the variance of the capital value growth. Equation (21) shows that the volatility of the direct real estate yields is an important component in explaining the direct real estate total risk changes. If changes in the direct real estate yields were always close to zero, then equation (21) implies that changes in real estate capital values would have scarcely any volatility.

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[8] Please refer to Appendix 2 for details on the derivation of this formula.

This current research paper utilizes the highly flexible Beta distribution for a specific direct real estate asset return, which is commonly used to represent the total-return variability over a fixed range. The Beta distribution is capable of capturing asymmetry as well as excess kurtosis, and can therefore readily proxy the direct commercial real estate return distribution. The probability density function for the Beta distribution is

$$\begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} W^{\alpha-1}(1-W)^{\beta-1} & 0<W<1; \\ 0 & \text{elsewhere} \end{cases}$$

Hence, expected utility can be represented below:

$$E[U(W)] = \int_0^1 U(W) \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} W^{\alpha-1}(1-W)^{\beta-1}\, dW$$

This equation in turn can be represented by a function $U(\alpha,\beta)$. Thus, the expected utility function $U(\alpha,\beta)$ is a valid representation of the investor's preferences toward uncertain rental income prospects through two parameters: the alpha and beta parameters of the beta probability density function. The mean and variance of the rental income distribution can be expressed as $\frac{\alpha}{\alpha+\beta}, \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ respectively.

The Integrated Direct Real Estate Risk Measure Model

This research is concerned with the rigorous integration of the bond duration-convexity, the Beta distribution function, and the real estate equivalent yield valuation conception to model the estimation of several key direct real estate risk measures on an *ex ante* basis (in the presence of limited information of a direct real estate asset). Such an integrated direct real estate risk-measure model is able to estimate the expected total returns for a direct real estate asset via a Beta distribution, for which a computerized sub-model is specially programmed. This integrated risk-measure model is depicted in Appendix 3 under a spreadsheet format, and the sub-model of the Beta distribution is denoted as the MATLAB program for the "Harry Potter" sub-model in Appendix 2. With limited and available information, for *e.g.*, from real estate consultants or valuers, pertaining to a specific direct real estate asset and its wider real estate market(s), the integrated risk-measure model deploys the concepts of duration and convexity, the LPM and the beta distribution, in order to estimate the expected total returns and capital values for the specific direct real estate asset. This integrated risk-measure model estimates several key measures, which include kurtosis, skewness, standard deviation, variance and value-at-risk.

The Beta Distribution

The Beta distribution is an integral part of the integrated direct real estate risk-measure model, which is utilized to generate a distribution of the total return for a specific direct real estate asset, on an *ex ante* basis, in the light of limited information. The Beta distribution for the specific direct real estate asset return is appropriate because the Beta distribution is a highly flexible distribution, commonly used to represent the total-return variability over a fixed range. In direct commercial real estate markets, specific factors related to the nature of the lease contract may well give rise to the skewness in the distributions of the commercial real estate returns. In particular, the use of long-term lease contracts in the commercial real estate market, typically with embedded upward only rent reviews, skews the payoffs associated with the holding commercial real estate.

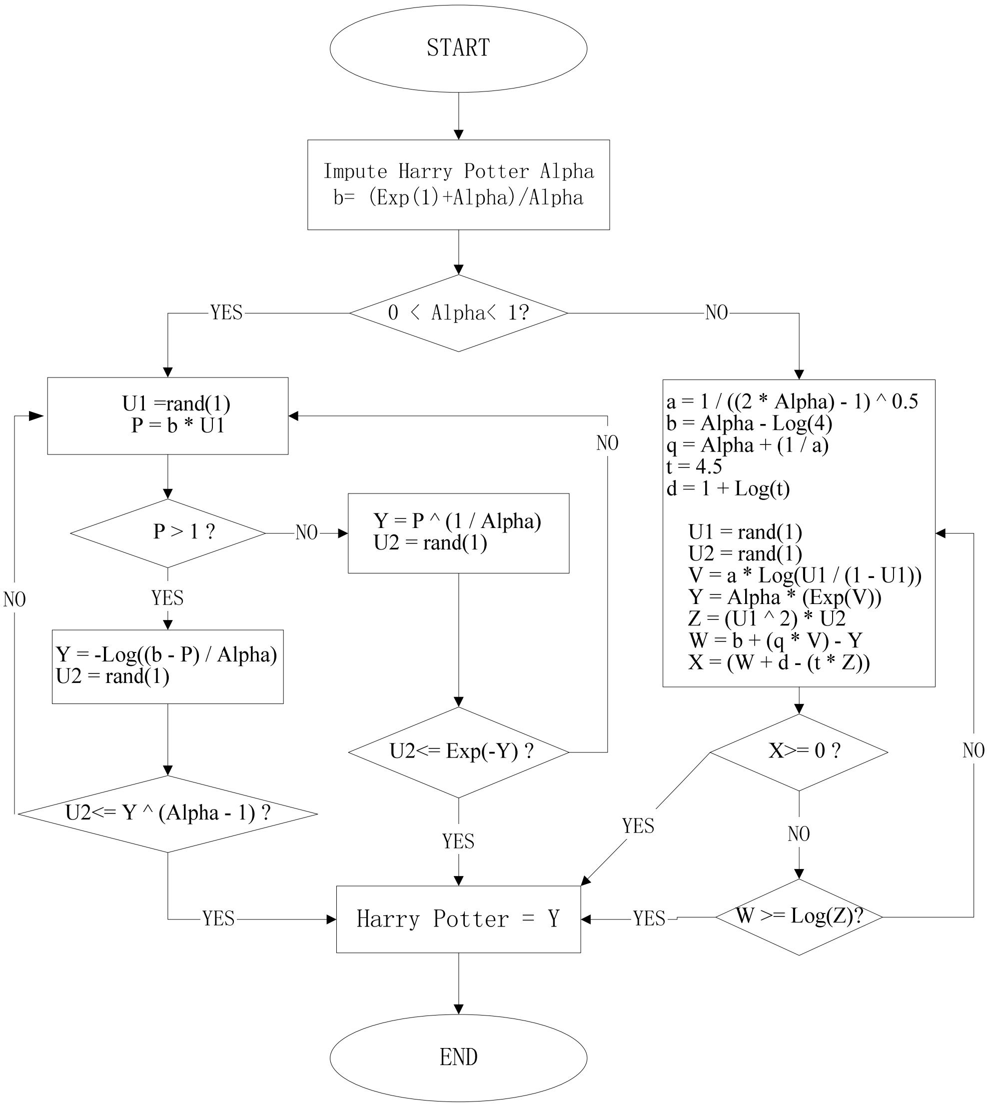
The Beta distribution is capable of capturing asymmetry as well as excess kurtosis, and thus can be readily used as the direct commercial real estate return distribution. Another consideration of this distribution is that the Beta distribution is cited in several research studies of total returns for the direct real estate assets in the UK and the US, Such as Lizieri and Ward (2001), Krystalogianni, A. and Tsolacos, S. (2004), and McDonald (1996). The beta distribution has the desirable feature that it can take on a wide variety of different shapes, yet is fully described by the values of only two parameters, the alpha and beta. Krystalogianni, A. and Tsolacos, S. (2004), in testing the normality of real estate return series of IPD data, apply The BestFit program (Palisade Corporation Copyright), which fits alternative distributions to frequency distributions and uses three tests to assess the goodness of fit of the theoretical distributions: the Chi-Square, the Kolmogorov-Smirnov and the Anderson–Darling test. All tests for normality reject the hypothesis that the normal distribution is an adequate fit of the observed returns; the Beta distribution appears the most plausible using all the tests. When the parameters are equal, the Beta distribution is symmetric. If either parameter has the value of 1 and the other being >1, the distribution is J-shaped. If alpha is smaller than beta, the distribution is positively skewered, and negatively skewered otherwise. Thus, the two main conditions underlying the Beta distribution are:

(a) The uncertain parameter, alpha, is a random value between 0 and a positive value;

(b) The shape of the distribution can be specified with the two positive parameter values for alpha and beta fixed.

The most common target outcomes of the parameters that can be derived from the limited information will be the minimum, maximum and most likely ones. Although the two parameter values that are required for the Beta distribution may not be readily available, or easily worked out, they can be estimated through a structured Monte Carlo simulation, but from an appropriate part of the integrated *ex-ante* risk-measure model that precludes the Beta distribution sub-model. The two parameter values for the Monte Carlo simulation are defined as "alpha 1" for alpha and "alpha 2" for beta. The alternative to the structured Monte Carlo simulation approach is the inclusion of the beta distribution sub-model which serves as an integral part of the integrated direct real estate risk-measure model. As depicted in Fig 2, the flow chart denotes the "Harry Potter" sub-model that represents the Beta distribution sub-model as a part and parcel of the direct real estate risk-measure (and return) model. The rectangles in Fig 2 represent the required computerized operations while the trapezoids represent the decision points. The "Harry Potter" sub-model program starts with the imputation of the two parameters, *Alpha* and *b*. It is required that $0<Alpha<1$, $0<b$, in order to satisfy the parameter requirements of the Beta distribution. In the first decision point, if $0<Alpha<1$ condition is not satisfied, the program will enter the “NO” sub-routine. An algorithm with several parameters, such as *a, b, q, t, d,* the random numbers *U1* and *U2* which are evenly distributed between 0 and 1*, V, Y, Z, W* and *X* will be carried out with decision points to make sure the conditions (such as *X*>=0, *W*>= Log(*Z*)) are satisfied in order obtain the estimated value of *Y* at the end of the program. If $0<Alpha<1$ condition is satisfied in the first decision point, the program will enter the “YES” sub-routine in the left side of the flow chart. Evenly distributed random number *U1* and parameter *P* will be assigned and an algorithm will be carried out with nested loops to make sure the conditions (such as *P*>1, *U2* <= *Y*^ (*Alpha-1*), and *U2* <= Exp (-*Y*)) are met in order to obtain the estimated value of *Y* at the end of the program. The “Harry Potter” sub-model is imperative because it generates a Gamma distribution for each of the two-parameter values, alpha and b. When the two distributions are combined, the beta distribution will be derived. Once the beta distribution is obtained, it will be rescaled in terms of the direct real estate yield (*y*), from which the duration and convexity are derived, and then used to provide the estimates of the direct real estate asset capital growth. The “Harry Potter” sub-model in MATLAB program and Visual Basic macro functionality format is attached in Appendix 2 for reference purposes.

Fig 2 The "Harry Potter" Sub-Model Flow Chart



(Sources: Author. 2009)

The Required Data Input Values

It is essential to clearly state at the onset the data input values required by the *ex ante* integrated direct real estate risk-measure model, on the basis of observed market conventions.

Equivalent (Rental) Yield

This equivalent (rental) yield is a valuation term, typically used in the UK and the British Commonwealth Countries, which defines the pro-rated annual interest gain from a direct real estate (investment) asset as a percentage of its current market price. It is properly defined in *eq* (13) and *eq* (14).

Riskless Return

This denotes the risk free rate of investment that is available if the investor is not inclined to take any form of risk. Thus, it is the minimum level of return that the investor expects before any form of investment is to be undertaken. In Singapore, the risk free rate is represented by treasury bonds offered by its central bank, the Monetary Authority of Singapore (MAS), in the range of 1.24% to 2.29% p.a. (for year 2002).

Target Return

This is the required rate of return set by an investor before undertaking any form of investment. The target return will always be higher than the riskless return as the investor factors the various forms of risk undertaken in investment into his target return. The risk types factored into the target return include the financial risk, the interest rate risk, the default risk and the prepayment risk among others. These risk types are deemed to be mutually exclusive, with no interactions between them. Each of these risks need to be taken into account and quantified, after which they were added in turn to the riskless return percentage to obtain the target return.

Expected (Rental) Market Yields

The expected (rental) yields denote the set of forecasts of the market yields for each year of the forecast period, under a real estate market analysis provided by a real estate consultant or a real estate market index service provider. The expected market yields are the expected levels of returns that the investor can expect, based on the anticipated market performance. The *ex ante* integrated direct real estate risk-measure model utilizes the expected (rental) market yields from the Jones Lang LaSalle Real Estate Intelligence Service-Asia (JLL REIS-Asia) data set.

The Integrated Risk-Measure Model Estimation

The model estimation of the *ex ante* integrated direct real estate risk-measure model utilized the JLL REIS-Asia data set for the prime Singapore and Hong Kong office sectors and in particular, the data for 2002. This data consists of the Raffles office market data and the Shenton office market data for the Singapore prime office sector; the Central office market data and the Wan Chai office market data for the Hong Kong prime office sector, which are presented in Appendix 4 for reference purposes.

Simulation Findings

For each set of market data, there will be firstly one structured Monte Carlo simulation model that is to be run per set. The simulation model's structure is represented by the *ex ante* integrated direct real estate risk-measure model but without the "Harry Potter" sub-model. For simulation purposes, the required inputs for the risk factors are presented in Appendix 4, in which these risk factors represent the set of limited information for the integrated risk-measure model.

Among the input risk factors in Appendix 4, the equivalent (rental) yield is taken to be synonymous to the prevailing initial (rental) yield as they both represent the percentage yield that an investor will likely gain from his investment in a real estate asset (for *e.g.*, an office building). Utilizing the Raffles Office market data for July 2002 as an example, the initial (annual) yield of 5.1% is obtained according to *eq* (7), by dividing the effective rent by the capital value:

$$\frac{\text{Effective Rent}}{\text{Capital Value}} = \text{Initial Yield}$$

$$\frac{\$604}{\$11{,}948} = 5.055\% \approx 5.1\%$$

However, the Effective Passing (Rental) Yield is at 5.7%, which implies that the actual rental income received per *sqm* is

$$\text{Effective Passing Yield} \times \text{Capital Value} = \text{Current (Rental) Income}$$

$$5.7\% \times \$11{,}948 \approx \$681 \text{ per sqm p. a.}$$

Therefore, the current (rental) income will actually be higher than the rental value. With the same calculations performed on the rest of the market data, the risk-factor input values that are entered into the structured Monte Carlo simulation-model are presented in Table 1 (actual calculations are detailed in Appendix 5). Rental Values will be the effective rents. The interest rate of the long-term government bonds is utilized as a proxy for the risk free rate (the riskless return) in Singapore. After accounting for the weighted difference between the 2-year bond yields and 5-year bond yield rates, the rate of riskless return for a 3-year investment will range from 1.24% to 2.29%. (Appendix 6)

Table 1 Market values to be imputed into the model

Prime Office Market	Equivalent Yield (%)	Current (Rental) Income ($psqm p. a.)	Rental Value ($psqm p. a.)	Target Return (%)[9]	Riskless Return (%)	Years to Review
Raffles (Singapore)	5.1 – 5.2	681 – 685	604 – 668	5.3%	1.24% - 2.29%	3
Shenton (Singapore)	5.4 – 5.6	533 - 551	490 – 540	6.4%	1.24% - 2.29%	3
Central (HK)	5.2 – 5.5	4,725 – 4,799	3,864 – 4,462	6.6%	2.38% – 4.61%	3
Wan Chai (HK)	4.0 – 4.2	2,871 – 2,957	2,191 – 2,420	6%	2.38% – 2.61%	3

(Source: Author, 2009; Crystal Ball program)

In Hong Kong, the government does not issue government bonds. However, since 1993, the Hong Kong dollar fixed income debt instruments-Exchange Fund Notes (EFNs) have been issued for the account of the Exchange Fund by the Hong Kong Monetary Authority (HKMA) under the Exchange Fund Ordinance, with an intention to replace the Government Bond Program with the Exchange Fund Note Program. The EFNs are utilized here as a proxy. As a 3-year period is included in the range of the time period for these EFNs, there is no need to calculate the weighted average. The rates for the 3-year EFNs are provided in Appendix 7.

The results of the simulations are presented in Appendix 8 to 10 for reference purposes; the simulation results are further listed in Table 2. From the sensitivity analysis in Table 2, it shows the Equivalent Yield of Raffles, Shenton, Central and Wan Chai will drop respectively -0.94% ~ -0.97% with 1% increase in the riskless rate; while the Rental Value for the 4 markets increase 0.18%-0.38% respectively. The modified duration's sensitivity is correctly negatively signed with respect to the equivalent value (EY),

[9] Values are based on Author's expert judgment, which accounts for economic conditions and types of risk mentioned in the earlier section of "*The Required Data Input Values*".

with a high sensitivity value range between -0.94~ -0.97; while, as expected, positively signed with respect to the rental value (RV), with the highest sensitivity values , 0.35 and 0.38, for the prime Singapore Shenton office market and the prime HK Central office market respectively.

Table 2 Summary of Modified Duration Simulation Results

Prime Office Market	Raffles (Singapre)	Shenton (Singapore)	Central (HK)	Wan Chai (HK)
Modified Duration (in yrs) (with 80% probability)	19.20 – 19.67	17.85 – 18.26	18.00 – 18.72	23.34 – 24.15
Modified Duration's Sensitivity Values with respect to EY and RV	EY (-0.97) RV (+0.18)	EY (-0.95) RV (+0.35)	EY (-0.94) RV (+0.38)	EY (-0.97) RV (+0.26)
Mean of the Modified Duration (in yrs)	19.43	18.05	18.36	23.74
Std. Deviation	**0.1**8	0.15	0.26	0.29
Variance	0.03	0.02	0.07	0.08
Skewness	0.00	0.07	0.02	0.09
Kurtosis	2.19	2.49	2.42	2.36

* EY = Equivalent Yield RV = Rental Value

(Source: Author, 2009; Crystal Ball program)

From the simulation results in Table 2, the modified duration shows a relatively symmetrical distribution for all the 4 office markets, with only a mild level of positive skewness in the range between 0.02 and 0.09. In contrast to the standard normal distribution's kurtosis of 3, the various distributions of the modified duration for the 4 markets are platykurtic with the corresponding kurtosis in the range between 2.19 and 2.49, implying a comparatively fatter distribution range.

Table 3 Summary of Simulation Results for Total Returns less than Target

Prime Office Market	Raffles (Singapore)	Shenton (Singapore)	Central (HK)	Wan Chai (HK)
TR < TaR (with 80% probability)	1.6% - 21.6%	0.0% - 25.4%	14.2% - 55.4%	56.0% – 93.8%
TR(<TaR)'s Sensitivity Values with respect to TaR and EY	TaR (+0.92) EY (-0.25)	TaR (+0.95) EY (-0.26)	TaR (+0.88) EY (-0.45)	TaR (+0.88) EY (-0.45)
Mean of the TR (<TaR)	9.7%	8.4%	33.7%	77.7%
Std. Deviation	8.4%	11.6%	15.8%	14.4%
Variance	0.7%	1.4%	2.5%	2.1%
Skewness	1.56	1.83	0.42	-0.77
Kurtosis	6.03	6.02	2.56	2.91

* TaR = Target Return TR=Total Return EY = Equivalent Yield

(Source: Author, 2009; Crystal Ball program)

Table 3 subsequently presents the simulation results pertaining to the total returns being less than the target return (*TaR*) with 80% probability, such as the variance, standard deviation and the relevant statistical tests. It is observed that the range for the standard deviation and variance concerning the total returns are very small, indicating that the dispersion of these values is mostly around the mean.

Values for the probability of the total returns less than target (with 80% probability) indicate greater differences among the 4 prime office markets. The prime Raffles office market appears to offer a relatively low risk-adjusted total return with a mean of 9.7% and a standard deviation of 8.4(an excess total return of 9.7% - 8.4% = 1.3% or 130 basis points). Positively skewed with a very high peak (leptokurtic) as indicated by the kurtosis of 6.03, implies that most of the distribution reflects minimum values that signify low probability. The prime Shenton office market offers a non risk adjusted total return with a mean total return of 8.4% that is well below the corresponding standard deviation of 11.6%. The Shenton office market has a more risky total return due to its higher standard deviation and variance values. Table 3 also shows the sensitivity of Total Return (TR) to the target rate of return. The TR for Raffles and Shenton will respectively increase 0.92% and 0.95% if the target rate of return increases 1%, while decrease respectively -0.29% and -0.26% with 1% increase in the Equivalent (rental) Yields.

However, from Table 3, both the prime Central and Wan Chai office markets in HK offer relatively high risk-adjusted total returns, with a 33.7% mean total return and a 15.8% standard deviation for Central; and with 77.7% mean total return and a 14.4% standard deviation for Wan Chai. Their total return distributions tend to be symmetrically distributed with a flatter peak (kurtosis values being 2.56 and 2.91), relative to the standard normal distribution's kurtosis of 3.0. Nevertheless, the prime Wan Chai office market has a negatively skewed value, indicating that its total return distribution tends towards the higher end of its positive total returns. The sensitivity analysis in Table3 shows that 1% increase of target rate of return will cause the total return (being less than the riskless return) of Central and Wan Chai to increase a same amount of 0.88%; while decrease -0.45% with 1% increase in the equivalent rental yields for Central and Wan Chai. It is readily presented that the total returns being less that the target return is positively related, as expected, to the target return (TaR), and with much high sensitivity values (between 0.88 and 0.92) for all the prime office markets. On the other hand, the sensitivity of total returns being less than TaR is correctly negatively signed with respect to the direct real estate asset equivalent rental yield, and a further relatively high and negative sensitivity value, -0.45, is found for the prime HK Central and Wan Chai office markets.

Table4 Results of the Probability of Total Returns < Riskless Returns from Simulation

Prime Office Market	Raffles (Singapore)	Shenton (Singapore)	Central (HK)	Wan Chai (HK)
TR < RR	0.4% – 3.2%	0.0 %– 0.0%	10.6%–26.4%	46.0%–70.0%
TR(<RR)'s Sensitivity Values with respect to EY and RR	EY (-0.81) RR (+0.25)	EY (-0.16) RR (+0.07)	EY (-0.87) RR (+0.36)	EY (-0.87) RR (+0.39)
Mean of the TR(<RR)	1.6%	0.0%	18.2%	57.9%
Std. Deviation	1.1%	0.1%	6.0%	9.1%
Variance	0.0%	0.0%	0.4%	0.8%
Skewness	0.94	6.48	0.34	0.02
Kurtosis	3.57	60.50	2.80	2.46

*EY = Equivalent (Rental) Yield TR = Total Return RR = Riskless Return

(Source: Author, 2009; Crystal Ball Program)

Table4 presents the summary of the simulation results for TRs less than the riskless return (the risk free rate). The Shenton office market is exceptional in offering a 0% TR, as reflected by the extreme positive skewness and kurtosis. The Raffles office market behaves similarly on the whole, indicating a low TR that is below the riskless return. However, the Central and Wan Chai office markets show much higher risk-adjusted TRs with 57.9% mean TR and 9.1% standard deviation for Wan Chai, and with 18.2% mean TR and 6.0% standard deviation for Central. As a result, Wan Chai will be attractive to the risk-taking real estate investors. Observed from the results in Table 4, the total returns being less than the riskless return (RR) are sensitive to the RR, and move in the same direction with it; while highly sensitive to the direct real estate equivalent rental yield (with sensitive values between -0.16~-0.87).

Findings of the complete ex ante Integrated Risk-measure Model with the "Harry Potter" Sub-Model

Finally, the complete and *ex ante* integrated direct real estate risk-measure model, inclusive of the "Harry

Potter” sub-model, is run for the data set of the 4 prime office markets in Singapore and HK. The model findings are presented in Table 5 and Table 6, from which it can be observed that the modified duration on the whole is highly sensitive to the equivalent yield (EY) and the Rental Value (RV). By marginally changing EY and then RV, while the rest of the model’s limited information set is kept constant, and *ceteris paribus*, then the impact on the modified duration can be readily observed in Table 5 and Table 6 (details are presented in Appendix 11 for reference purpose).

Table 5 Sensitivity Results on Modified Duration with Marginal Change in the Equivalent Yield

Prime Office Market	Initial Equivalent Yield	+1%	-1%
Raffles (Singapore)	5.1%	6.1%	4.1%
Modified Duration(in yr)	19.43	16.22 (-16.52%)*	24.20 (24.55%)
Shenton (Singapore)	5.5%	6.5%	4.5%
Modified Duration (in yr)	18.06	15.26 (-15.50%)	22.09 (22.31%)
Central (HK)	5.4%	6.4%	4.4%
Modified Duration (in yr)	18.18	15.30 (-15.84%)	22.37 (23.05%)
Wan Chai (HK)	4.1%	5.1%	3.1%
Modified Duration (in yr)	23.74	18.98 (-20.05%)	31.57 (32.98%)

* All numbers in brackets are the percentage of change in the Modified Duration.
(Source: Author, 2009)

In the three prime office markets of Raffles, Shenton and Central, Table 5 shows that an increase in 1 percentage point in the equivalent yield (EY) will result in an approximate decrease of 15.50% to 16.52% to the modified duration, while a much lager decrease of 20.05% for the Wan Chai office market. When the Equivalent Yield drops by 1 percentage point, the modified duration increases from 23.05% to 24.55% for the first 3 office markets of Raffles, Shenton and Central, while the Wan Chai office market shows a highly significant increase of 32.98%.

It can be readily observed in Table 6 that for all the 4 office markets in general, a 10% increase on the initial rental value (RV) will effectively result in a 1.18% to 1.32% increase in the modified duration in year terms, while a 10% decrease on the initial RV will result in a 1.43% to 1.65% decrease in the modified duration in year terms.

Table 6 Sensitivity Results on Modified Duration with Marginal Change in the Rental Value (RV)

Prime office market	Initial Rental Value	+10% change in RV	-10% change in RV
Raffles (Singapre)	S$636	S$700	S$572
Modified Duration (in yrs)	19.43	19.67 (1.24%)*	19.14 (-1.49%)
Shenton (Singapore)	S$515	S$567	S$463
Modified Duration (in yrs)	18.06	18.29 (1.27%)	17.78 (-1.55%)
Central (HK)	HK$4,163	HK$4,580	HK$3,746
Modified Duration (in yrs)	18.18	18.42 (1.32%)	17.88 (-1.65%)
Wan Chai (HK)	HK$2,306	HK$2,437	HK$2,075
Modified Duration (in yrs)	23.74	24.02 (1.18%)	23.40 (-1.43%)

*All numbers in the brackets are the percentage of change in the modified duration.
(Source: Author, 2009)

From Table 7, the corresponding LPM risk measures are presented where the 3rd-order LPM for the risk-averse investor is estimated to be relatively high 0.185, while the 0.5th-order LPM for the aggressive investor is very low at 0.012. The associated reward (return) per unit of LPM risk is very low at about 0.52 for the direct real estate risk-averse investor; while, as expected, it is very high at about 8.24 for the aggressive direct real estate asset investor.

Table 7 Risk Estimates via Low Partial Moment (LPM) Approach

LPM risk Preference	**Aggressive** **Averse**			
The order of LPM	0.50	1.00	2.00	3.00
LPM Risk	0.012	0.031	0.081	0.185
Return/LPM (%)	8.241	3.104	1.166	0.515

(Source: Author; 2009)

Concluding Comments

Research four of this research demonstrates that in the presence of a set of limited available information comprising a direct real estate asset's passing (annual) rent, the current rental value, the expected yields and the yield-growth movements from a real estate market analysis conducted by a real estate consultancy or service provider, the risk-free rate and the lease maturity period, it is readily feasible to model and rigorously estimate several key risk measures as well as the expected returns. They can be achieved through an *ex ante* integrated direct real estate risk-measure model that innovatively combines the bond duration-convexity risk conception, the Beta distribution function and the real estate equivalent (rental) yield valuation conception. The integrated risk-measure model findings, conducted under the structured Monte Carlo simulation but without the Beta distribution sub-model, the "Harry Potter" computable program, will suggest that higher risks do not necessarily result in higher total returns. Although the levels of total return among the four prime office markets of Raffles (Singapore), Shenton (Singapore), Central (HK) and Wan Chai (HK), do differ slightly; the associated level of risk appears to differ to a greater extent. Wan Chai (HK) has the highest duration value of 23.7 years and the lowest equivalent yield, while Shenton (Singapore) has the lowest duration value of 18.1 years and the highest equivalent yield. However, in both markets, the levels of targeted total returns do not differ greatly.

Upon a sensitivity analysis, the complete and *ex ante* integrated direct real estate risk-measure model, which incorporates the "Harry Potter" sub-model program, suggests that the equivalent (rental) yield is the most significant risky input factor affecting the modified duration and hence, the expected total return. This implies that this equivalent (rental) yield is more important a factor for risk-averse investors to form an expectation of the equivalent yield of a direct real estate asset, within the wider context of the real estate market yield. The distinct advantage of the complete and *ex ante* integrated risk-measure model over other traditional models for the direct real estate risk measures is that no past time-series data is involved. Such a rigorous model can readily model and estimate the key risk measures and the expected returns of the new direct real estate assets, which do not have historical data. In addition, the resulting model estimation of several key risk measures into the *ex ante* integrated model enables the user of the model to compare the model results with actual performance, as it unfolds over time.

In this research, the *ex ante* integrated direct real estate risk-measure model is merely applied to the prime office sector. Further investigative research can be carried out in the other real estate sectors, such as the industrial or commercial sectors to estimate the distribution of total returns and risks. This section concentrates on the concepts of the direct real estate duration, convexity and the Beta distribution; some other appropriate distributions for the expected direct real estate returns and risk can be attempted in a future study.

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Appendices

Appendix 1. <u>The Proof for *eq* (17)</u>

Based upon the equivalent yield model for a direct real estate asset *j*

$$V_{jt} = \frac{a_{jt}}{y_{jt}} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n} \qquad (14)$$

the first derivation of the capital value, $V_{jt,}$ with respect to y_{jt} can be defined in eq (15) as

$$\frac{dV_{jt}}{dy_{jt}} = -\frac{a_{jt}}{y^2{}_{jt}} - \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n}\left[\frac{1}{y_{jt}} - \frac{n}{(1+y_{jt})}\right] \qquad (15)$$

Further, rearranging eq (14) produces

$$V_{jt} = \frac{a_{jt}}{y_{jt}} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n} = \frac{a_{jt}(1+y_{jt})^n + (RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n}$$

hence,

$$\frac{1}{V_{jt}} = \frac{y_{jt}(1+y_{jt})^n}{a_{jt}(1+y_{jt})^n + (RV_{jt} - a_{jt})}$$

In combination with eq (12),

$$-D_{jt}* = \frac{dV_{jt}}{dy_{jt}} \times \frac{1}{V_{jt}} \qquad (12)$$

multiplying $\frac{1}{V_{jt}}$ on the both sides of eq (15) will produce the modified duration, $D_{jt}*$, for the direct real estate asset j, as shown in eq (16).

$$D^*_{jt} = \left\{ \frac{a_{jt}}{y^2_{jt}} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1+y_{jt})^n} \left[\frac{1}{y_{jt}} - \frac{n}{(1+y_{jt})} \right] \right\} \bullet \frac{y_{jt}(1+y_{jt})^n}{a_{jt}(1+y_{jt})^n + (RV_{jt} - a_{jt})} \qquad (16)$$

In the instance of a fully rack-rented real estate asset where the direct real estate rental value, RV_{jt}, is equal to the passing annual rental income, a_{jt}, then the modified duration, D^*_{jt}, can be further simplified.

$$D^*_{jt} = \left\{ \frac{a_{jt}}{y^2_{jt}} \right\} \bullet \frac{y_{jt}(1+y_{jt})^n}{a_{jt}(1+y_{jt})^n} = \frac{1}{y_{jt}} \qquad (17)$$

Appendix 2. The MATLAB program for the "Harry Potter" Sub-Model

```
Function HarryPotter(Alpha)

b = (Exp(1) + Alpha) / Exp(1)
If Alpha > 0 And Alpha < 1 Then GoTo Step1A
If Alpha > 1 Then GoTo Step3A

Step1A:
  U1 = Rnd()
  P = b * U1
    If P > 1 Then GoTo Step2A

    Y = P ^ (1 / Alpha)
    U2 = Rnd()

    Select Case U2
      Case Is <= Exp(-Y)
      GoTo Finish
    End Select
    GoTo Step1A


Step2A:

  Y = -Log((b - P) / Alpha)
  U2 = Rnd()

    Select Case U2
      Case Is <= Y ^ (Alpha - 1)
      GoTo Finish
    End Select
    GoTo Step1A



Step3A:
a = 1 / ((2 * Alpha) - 1) ^ 0.5
b = Alpha - Log(4)
q = Alpha + (1 / a)
t = 4.5
d = 1 + Log(t)

  U1 = Rnd()
  U2 = Rnd()
  V = a * Log(U1 / (1 - U1))
  Y = Alpha * (Exp(V))
  Z = (U1 ^ 2) * U2
  W = b + (q * V) - Y
  X = (W + d - (t * Z))

  Select Case X
    Case Is >= 0
```

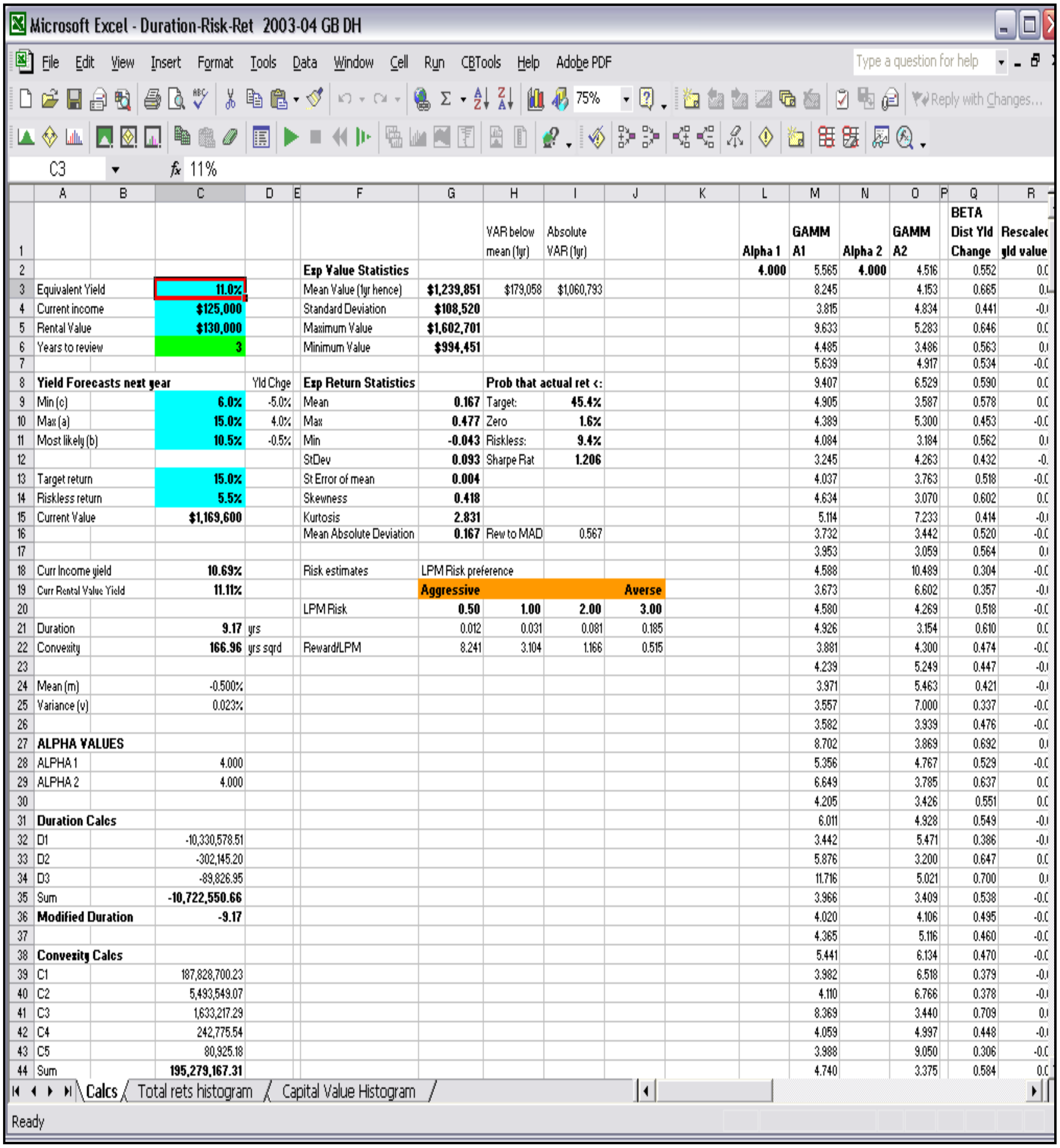
```
            GoTo Finish
            End Select
        GoTo Step4A


Step4A:
        Select Case W
            Case W >= Log(Z)
            GoTo Finish
            End Select

        GoTo Step3A

Finish:
    HarryPotter = Y
End Function
```

Appendix 3. **Screen shot of the main aspects of the Duration-Risk Model**



Microsoft Excel - Duration-Risk-Ret 2003-04 GB DH

C3 f_x 11%

	A	C	D	F	G	H	I	J	L	M	N	O	Q	R
1						VAR below mean (1yr)	Absolute VAR (1yr)		**Alpha 1**	**GAMM A1**	**Alpha 2**	**GAMM A2**	**BETA Dist Yld Change**	**Rescaled yld value**
2				**Exp Value Statistics**					**4.000**	5.565	**4.000**	4.516	0.552	0.0
3	Equivalent Yield	**11.0%**		Mean Value (1yr hence)	**$1,239,851**	$179,058	$1,060,793			8.245		4.153	0.665	0.
4	Current income	**$125,000**		Standard Deviation	**$108,520**					3.815		4.834	0.441	-0.
5	Rental Value	**$130,000**		Maximum Value	**$1,602,701**					9.633		5.283	0.646	0.0
6	Years to review	**3**		Minimum Value	**$994,451**					4.485		3.486	0.563	0.
7										5.639		4.917	0.534	-0.0
8	**Yield Forecasts next year**		Yld Chge	**Exp Return Statistics**		**Prob that actual ret <:**				9.407		6.529	0.590	0.0
9	Min (c)	**6.0%**	-5.0%	Mean	**0.167**	Target:	**45.4%**			4.905		3.587	0.578	0.0
10	Max (a)	**15.0%**	4.0%	Max	**0.477**	Zero	**1.6%**			4.389		5.300	0.453	-0.0
11	Most likely (b)	**10.5%**	-0.5%	Min	**-0.043**	Riskless:	**9.4%**			4.084		3.184	0.562	0.
12				StDev	**0.093**	Sharpe Rat	**1.206**			3.245		4.263	0.432	-0.
13	Target return	**15.0%**		St Error of mean	**0.004**					4.037		3.763	0.518	-0.0
14	Riskless return	**5.5%**		Skewness	**0.418**					4.634		3.070	0.602	0.0
15	Current Value	**$1,169,600**		Kurtosis	**2.831**					5.114		7.233	0.414	-0.
16				Mean Absolute Deviation	**0.167**	Rew to MAD	0.567			3.732		3.442	0.520	-0.0
17										3.953		3.059	0.564	0.
18	Curr Income yield	**10.69%**		Risk estimates	LPM Risk preference					4.588		10.489	0.304	-0.0
19	Curr Rental Value Yield	**11.11%**			**Aggressive**			**Averse**		3.673		6.602	0.357	-0.
20				LPM Risk	**0.50**	**1.00**	**2.00**	**3.00**		4.580		4.269	0.518	-0.0
21	Duration	**9.17**	yrs		0.012	0.031	0.081	0.185		4.926		3.154	0.610	0.0
22	Convexity	**166.96**	yrs sqrd	Reward/LPM	8.241	3.104	1.166	0.515		3.881		4.300	0.474	-0.0
23										4.239		5.249	0.447	-0.
24	Mean (m)	-0.500%								3.971		5.463	0.421	-0.
25	Variance (v)	0.023%								3.557		7.000	0.337	-0.0
26										3.582		3.939	0.476	-0.0
27	**ALPHA VALUES**									8.702		3.869	0.692	0.
28	ALPHA 1	4.000								5.356		4.767	0.529	-0.0
29	ALPHA 2	4.000								6.649		3.785	0.637	0.0
30										4.205		3.426	0.551	0.0
31	**Duration Calcs**									6.011		4.928	0.549	-0.
32	D1	-10,330,578.51								3.442		5.471	0.386	-0.
33	D2	-302,145.20								5.876		3.200	0.647	0.0
34	D3	-89,826.95								11.716		5.021	0.700	0.
35	Sum	**-10,722,550.66**								3.966		3.409	0.538	-0.0
36	**Modified Duration**	**-9.17**								4.020		4.106	0.495	-0.0
37										4.365		5.116	0.460	-0.0
38	**Convexity Calcs**									5.441		6.134	0.470	-0.0
39	C1	187,828,700.23								3.982		6.518	0.379	-0.
40	C2	5,493,549.07								4.110		6.766	0.378	-0.
41	C3	1,633,217.29								8.369		3.440	0.709	0.
42	C4	242,775.54								4.059		4.997	0.448	-0.
43	C5	80,925.18								3.988		9.050	0.306	-0.0
44	Sum	**195,279,167.31**								4.740		3.375	0.584	0.0

Calcs / Total rets histogram / Capital Value Histogram

Ready